Avalanches

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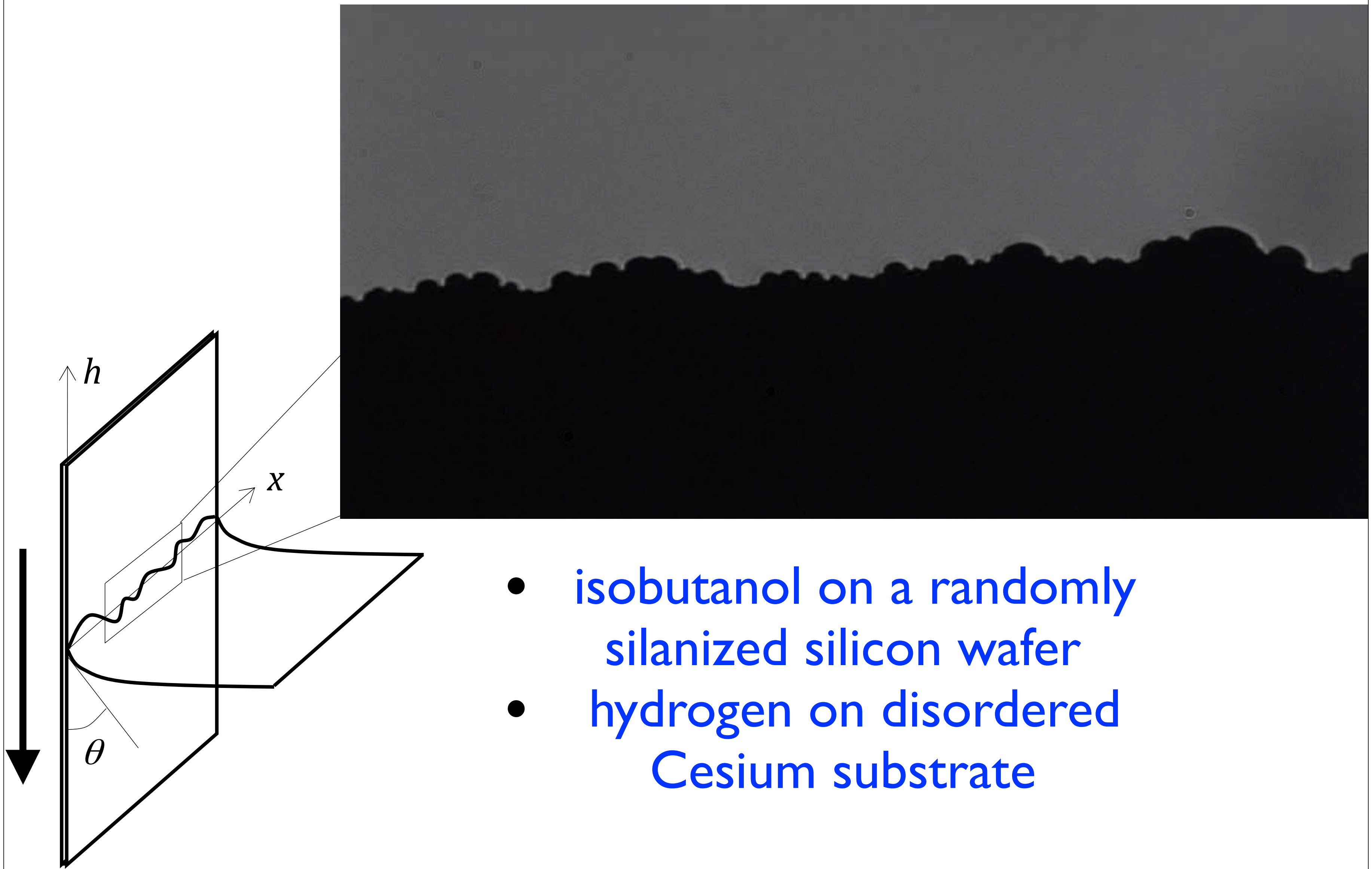
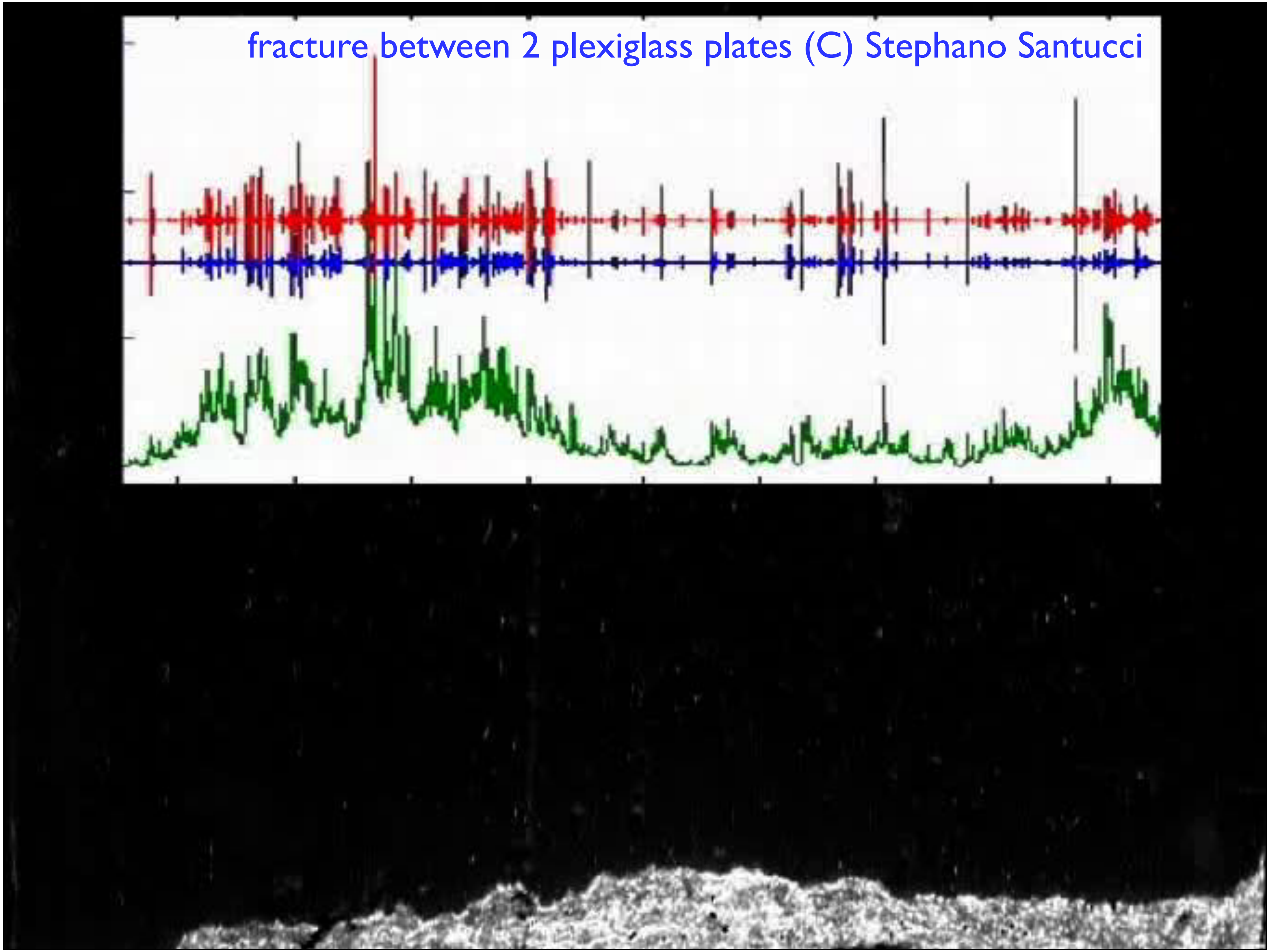
Grenoble, June 2014

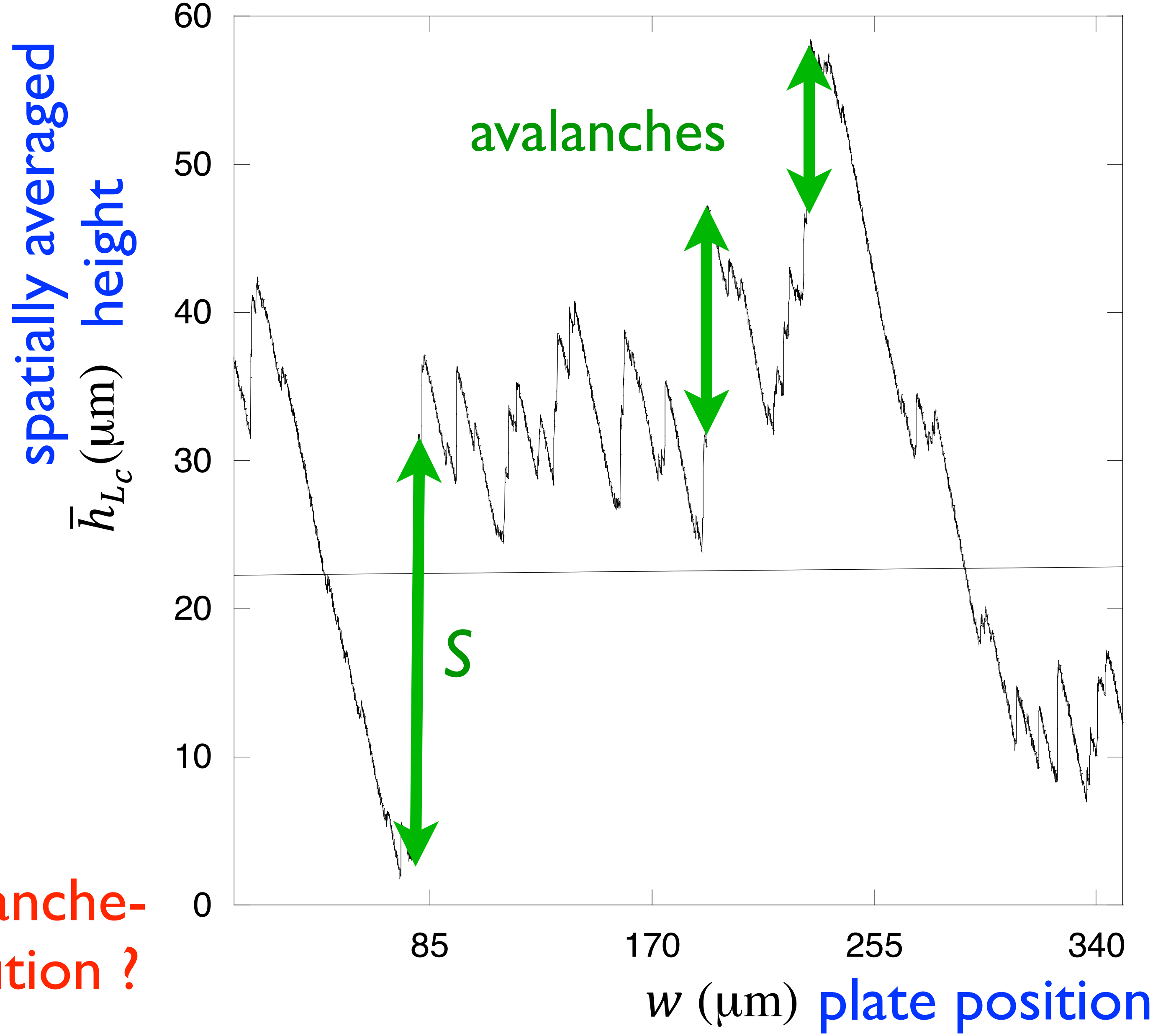
Contact line wetting



- isobutanol on a randomly silanized silicon wafer
- hydrogen on disordered Cesium substrate


fracture between 2 plexiglass plates (C) Stephano Santucci

height jumps = avalanches



what is avalanche-size distribution ?

The model

Displacement field $\quad x \in \mathbb{R} \longrightarrow u(x) \in \mathbb{R}$

Elastic energy:
$$\mathcal{H}_{\text{el}} = \frac{1}{2}\int \frac{\mathrm{d}^d k}{2\pi} \left|\tilde{u}_k\right|^2 \varepsilon_k + \int_x \frac{m^2}{2}\left[u(x) - w\right]^2$$
$$w = vt$$

for contact angle $\theta = 90^\circ$: $\quad \varepsilon_k \approx \sqrt{k^2 + \kappa^2} - \kappa$

$\kappa^{-1} = m^{-2}$ kapillary length $\quad$ (instead of $\varepsilon_k = k^2$)

Disorder energy
$$\mathcal{H}_{\text{DO}} = \int \mathrm{d}^d x\, V(x, u(x))$$

with correlations
$$\overline{V(x,u)V(x',u')} = \delta^d(x - x')R(u - u')$$

Functional renormalization group (FRG)

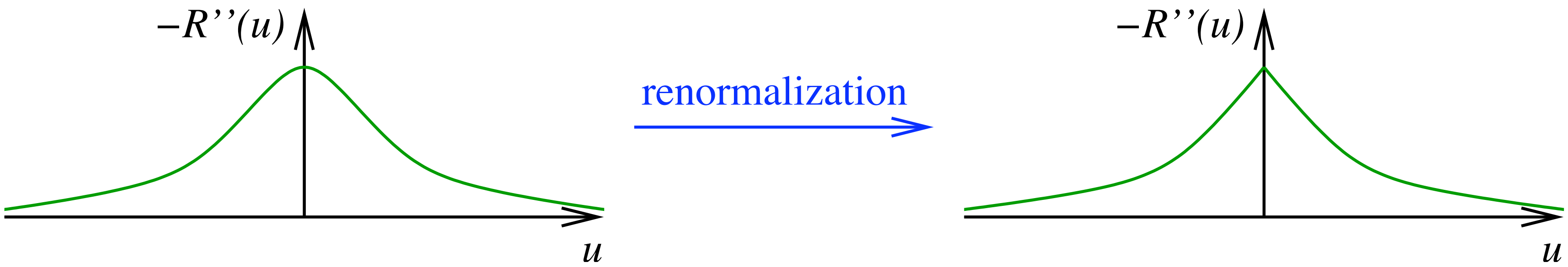
(D. Fisher 1986)

$$\frac{\mathscr{H}[u]}{T} = \frac{1}{2T}\sum_{\alpha=1}^{n}\left[\int_k \varepsilon_k |\tilde{u}_k^\alpha|^2 + \int_x m^2 (u^\alpha(x) - w)^2\right]$$
$$-\frac{1}{2T^2}\int_x \sum_{\alpha,\beta=1}^{n} R\big(u^\alpha(x) - u^\beta(x)\big)$$

Functional renormalization group equation (FRG) for the disorder correlator $R(u)$ at 1-loop order:

$$-\frac{m\mathrm{d}}{\mathrm{d}m}R(u) = (\varepsilon - 4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$

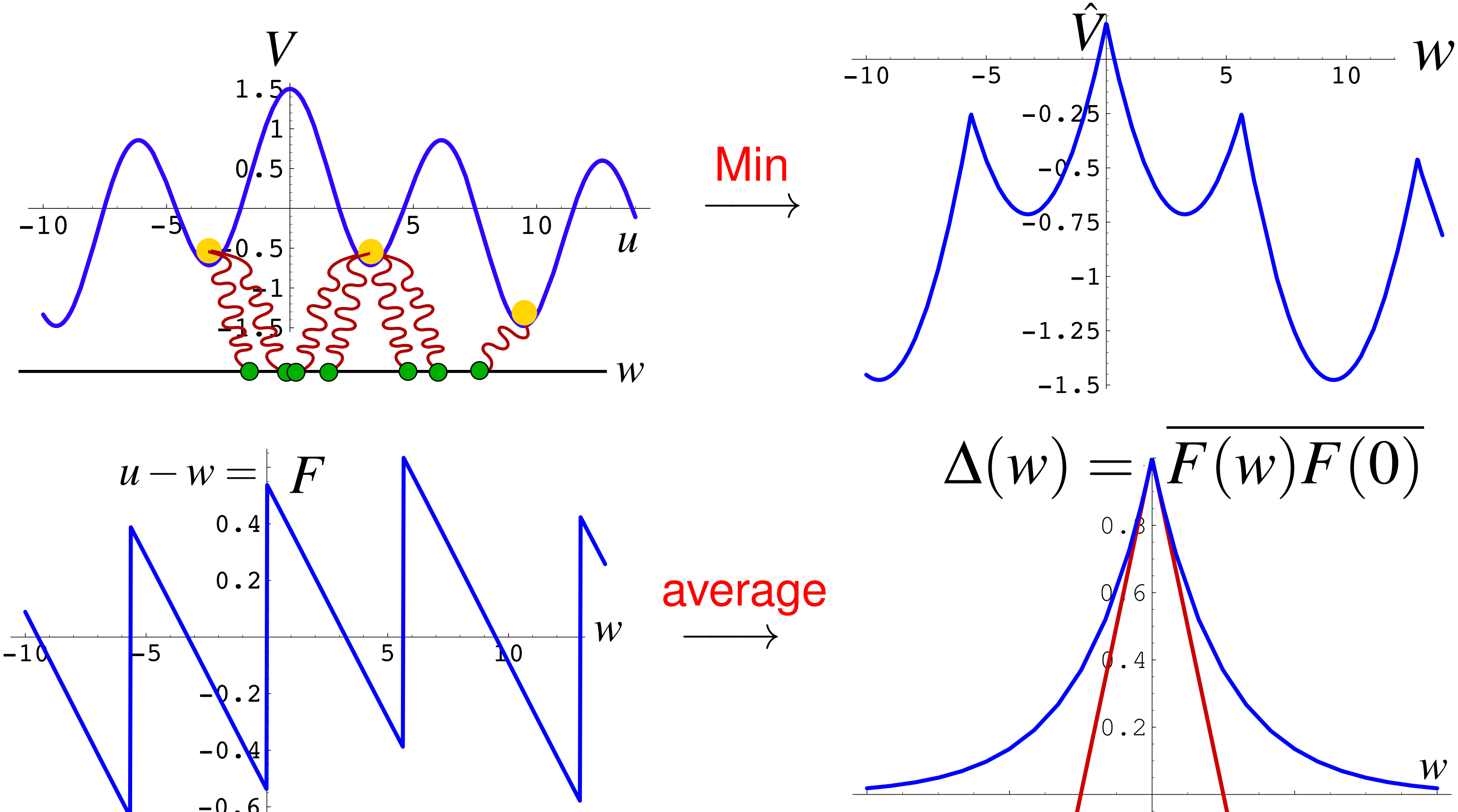
Solution for force-force correlator $-R''(u)$:



Cusp: $R''''(0) = \infty$ appears after finite RG-time (at Larkin-length)

Why is a cusp necessary?

...calculate effective action for single degree of freedom...

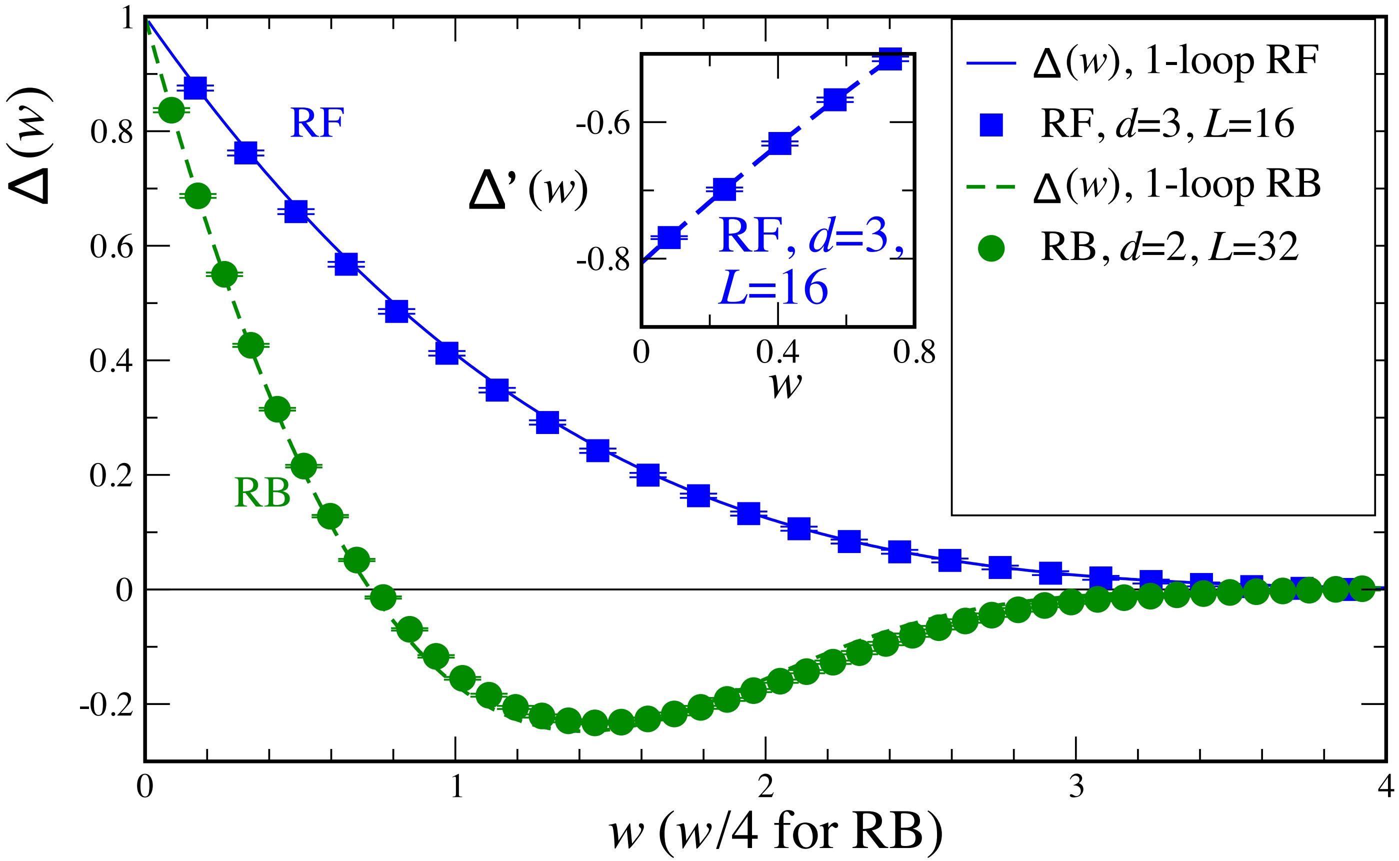


$$\Delta(w) = \overline{F(w)F(0)}$$

$$\overline{[F(w) - F(0)]^2} = \underbrace{\rho_{\text{shock}}|w|}_{p_{\text{shock}}} \langle S^2 \rangle$$

Measuring the cusp = effective action

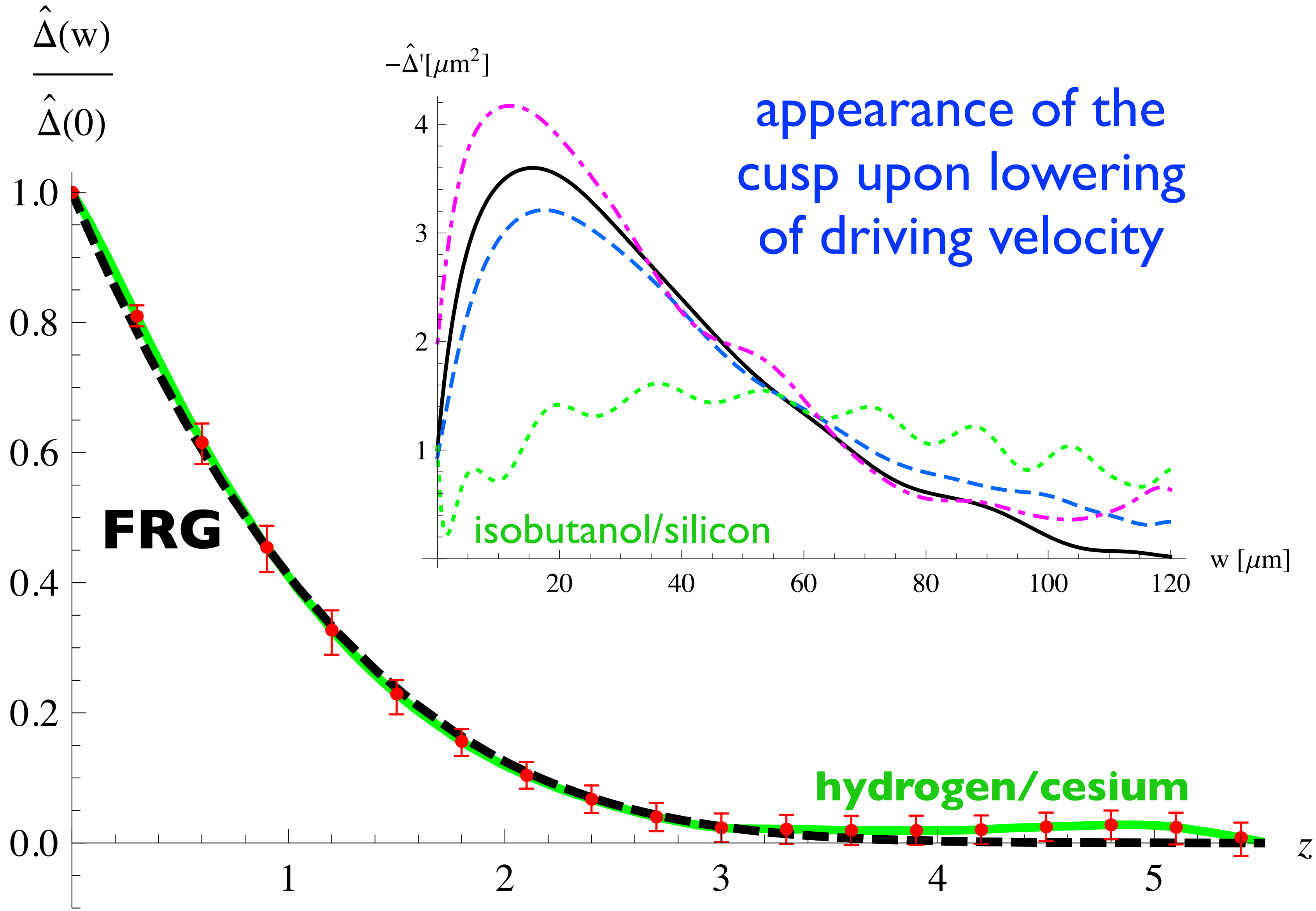
PLD+KW+A. Middleton



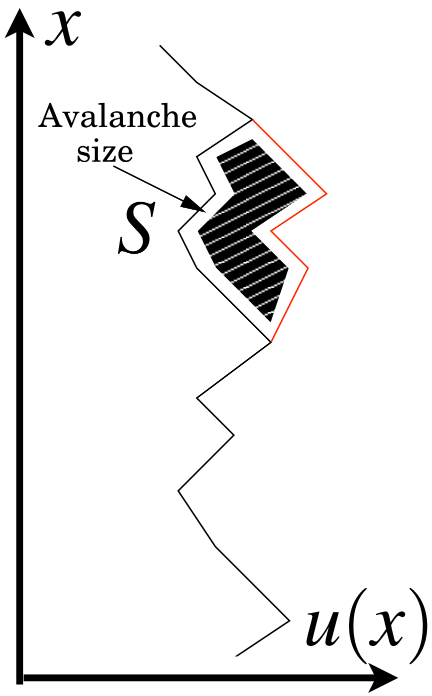
$$\Delta(w - w') = m^4 L^d \overline{[u_w - w]\,[u_{w'} - w']}$$

Δ = renormalized disorder correlator

The renormalized force-force correlator



Slope at the cusp and avalanche size moments



$$\rho \left\langle S \right\rangle |w - w'| = L^d \overline{|u_w - u_{w'}|} = L^d |w - w'|$$

#avalanches/unit length

$$\rho \left\langle S^2 \right\rangle |w - w'| \approx L^{2d} \overline{|u_w - u_{w'}|^2}$$

$$\approx 2L^d \frac{|\Delta'(0^+)|}{m^4} |w - w'|$$

together:
(exact)

$$S_m := \frac{\left\langle S^2 \right\rangle}{2 \left\langle S \right\rangle} = \frac{|\Delta'(0^+)|}{m^4}$$

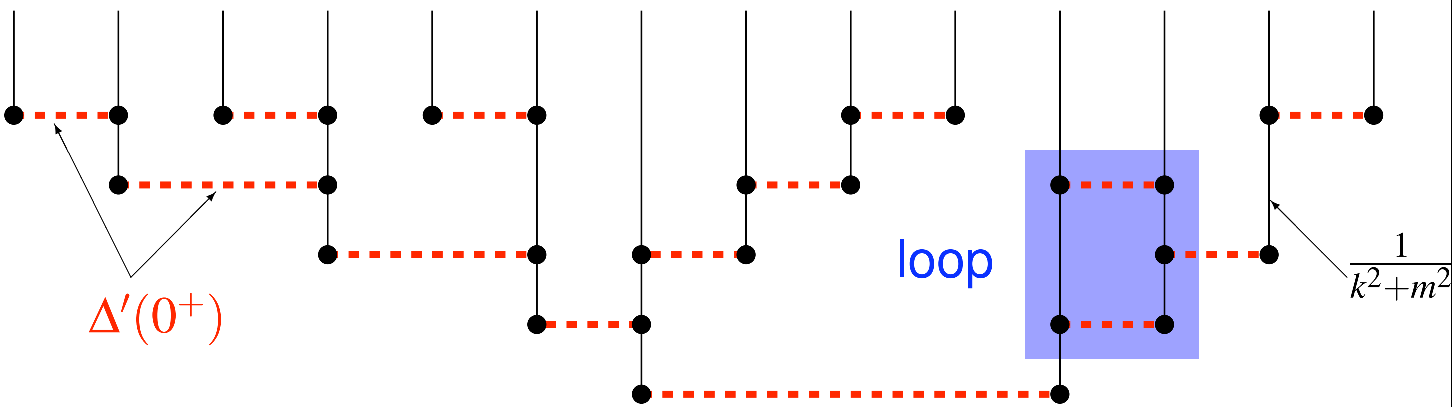
FRG-calculation

calculate the generating function $Z(\lambda)$ of avalanche-sizes S:

$$Z(\lambda) = \frac{1}{\langle S \rangle}\Big(\langle e^{\lambda S}\rangle - 1 - \lambda\langle S\rangle\Big)$$

$$\overline{e^{\lambda[u(w)-w-u(0)]}} - 1 = Z(\lambda)w + O(w^2) \quad \text{for} \ \ w>0\ .$$

$$Z(\lambda) = \sum$$

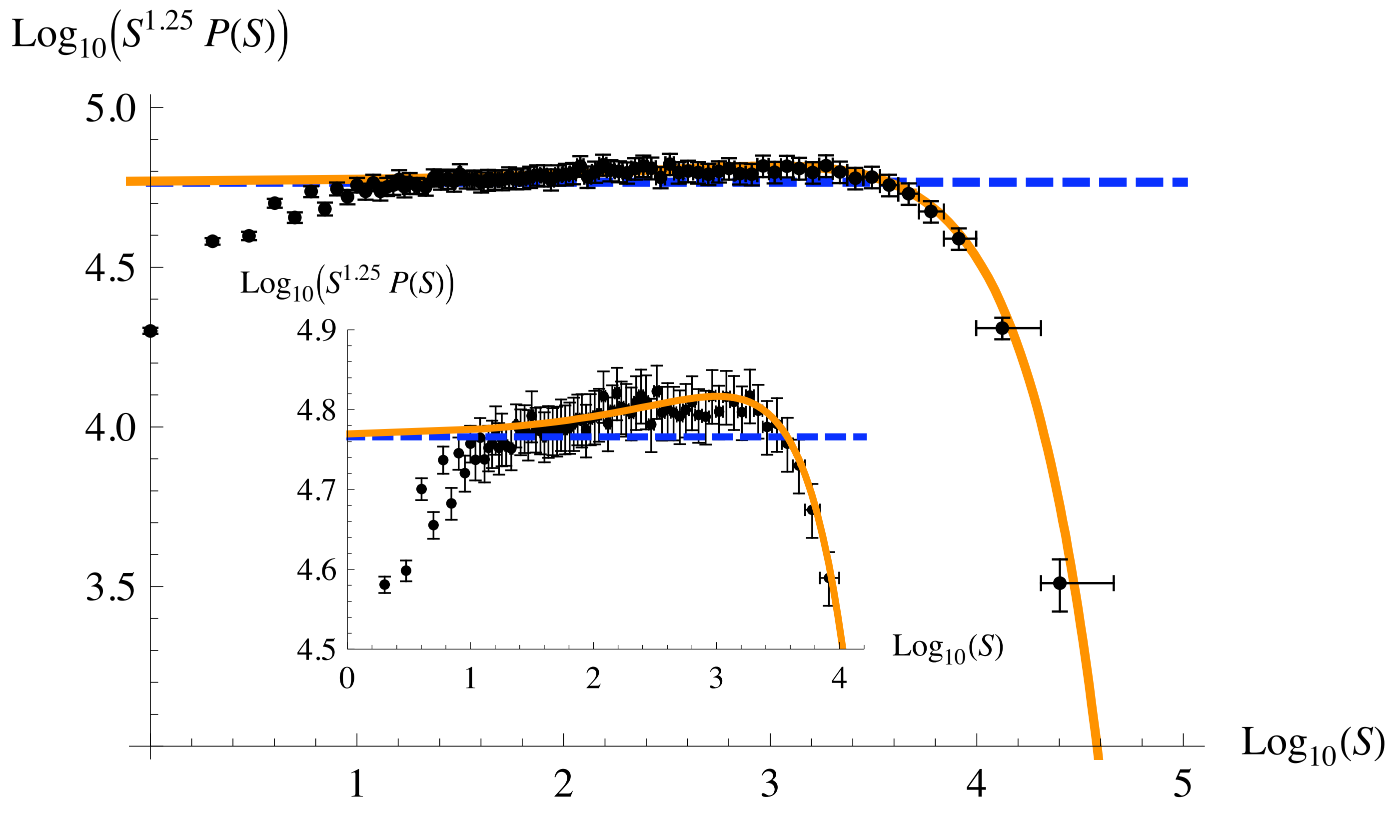


$$\Delta(w) - \Delta(0) \longrightarrow \Delta'(0^+)w + \ldots$$

Recursion Relation:

$$Z(\lambda) = \lambda - \underbrace{\Delta'(0^+)Z(\lambda)^2}_{\text{trees}} + \frac{\Delta''(0)}{\Delta'(0^+)} \sum_{n\geq 3}(n+1)2^{n-2} \underbrace{\int_k \frac{[-\Delta'(0^+)\,Z(\lambda)]^n}{(k^2+1)^n}}_{\text{loops with } n \text{ outgoing legs}}\ ,$$

Avalanche distribution



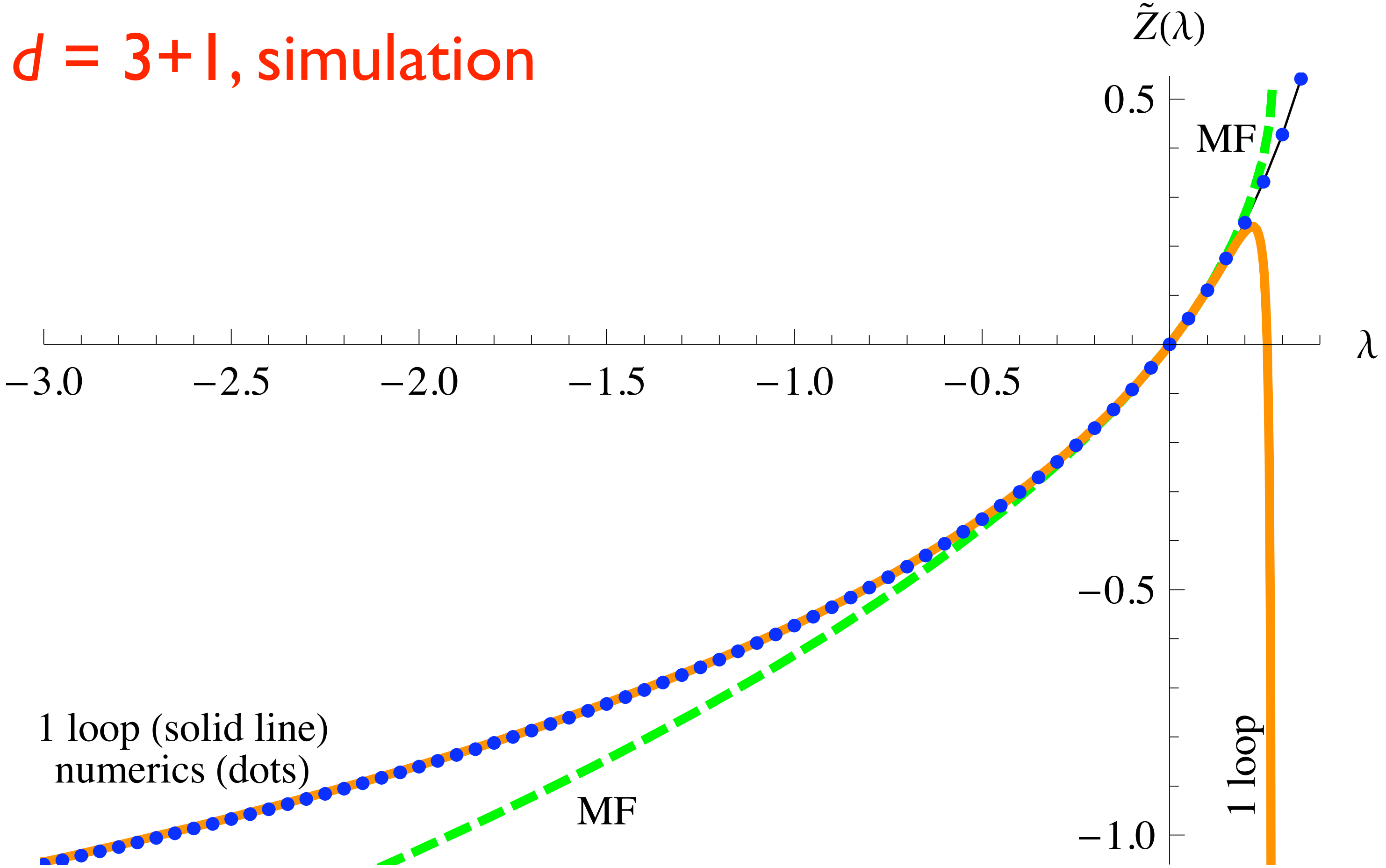
$$P(S) = \frac{\langle S\rangle}{2\sqrt{\pi}} S_m^{\tau-2} A S^{-\tau} \exp\left(C\sqrt{\frac{S}{S_m}} - \frac{B}{4}\left[\frac{S}{S_m}\right]^{\delta}\right)$$

$$\tau = \frac{3}{2} - \frac{1}{8}(\varepsilon-\zeta) + \ldots$$

$$\delta = 1 + \frac{1}{4}(\varepsilon-\zeta) + \ldots$$

Numerical results for RF interface in $d = 3+1$
P. Le Doussal, A. Middleton, KW
arXiv:0803.1142

RF, d = 3+1, simulation



$$Z(\lambda) = \overbrace{\frac{1}{2}\left[1-\sqrt{1-4\lambda}\right]}^{\text{MF = trees}} \underbrace{-\frac{\Delta''(0)}{4\sqrt{1-4\lambda}}\left[\log(1-4\lambda)(3\lambda+\sqrt{1-4\lambda}-1)-2(2\lambda+\sqrt{1-4\lambda}-1)\right]}_{\text{1 loop}} + \dots$$

Velocity distribution in an avalanche

classical Langevin equation

$$\eta \partial_t u(x,t) = \nabla^2 u(x,t) + m^2 \left[w - u(x,t)\right] + F(x, u(x,t))$$

this is now a theory of the velocity, not of the position:

$$S = \int_{x,t} \tilde{u}(x,t) \Big[\eta \partial_t \dot{u}(x,t) - \nabla^2 \dot{u}(x,t) + m^2 \big(\dot{w} - \dot{u}(x,t)\big) \Big]$$

$$- \int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t') \partial_t \partial_{t'} \Delta\big(u(x,t) - u(x,t')\big)$$

Disorder Vertex:

$$\begin{aligned} &\partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt}) \partial_{t'} \Delta'(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt}) \Delta'(0^+) \partial_{t'} \mathrm{sgn}(t-t') + \ldots \end{aligned}$$

simplifies to

$$S_{\mathrm{dis}}^{\mathrm{tree}} = \Delta'(0^+) \int_{xt} \tilde{u}_{xt} \tilde{u}_{xt} (v + \dot{u}_{xt})$$

!!! simple local cubic theory !!!

Avalanche Instanton

If $\lambda(x,t) = \lambda\,\delta(t)$ then the instanton equation is

$$(\partial_t - 1)\tilde{u}_t + \tilde{u}_t^2 = -\lambda\delta(t)$$

Solution

$$\tilde{u}_t = \frac{\lambda}{\lambda + (1-\lambda)e^{-t}}\theta(-t)$$

$$Z_{\text{tree}}(\lambda) = \left\langle \mathrm{e}^{\lambda \dot{u}(t)} - 1 \right\rangle\Big|_{t=0} = \int_{t<0} \tilde{u}_t = -\ln(1-\lambda)$$

$$\mathscr{P}_{\text{tree}}(\dot{u}) = \frac{\mathrm{e}^{-\dot{u}}}{\dot{u}}$$

MF
= ABBM
for COM
observables

higher-point functions also possible.

Scaling laws

suppose that there is a small-m limit of response to kick

$$\lim_{m\to 0} \frac{\delta u(x,t)}{\delta f} = \text{finite} \Leftrightarrow \tilde{u}(x,t) \text{ unrenormalized}$$

This implies a plethora of scaling laws:

	$\mathscr{P}(S)$	$\mathscr{P}(S_\phi)$	$\mathscr{P}(T)$	$\mathscr{P}(\dot{u})$	$\mathscr{P}(\dot{u}_\phi)$
	$S^{-\tau}$	$S_\phi^{-\tau_\phi}$	$T^{-\alpha}$	$\dot{u}^{-\mathsf{a}}$	$\dot{u}_\phi^{-\mathsf{a}_\phi}$
SR	$\tau = 2 - \frac{2}{d+\zeta}$	$\tau_\phi = 2 - \frac{2}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-2+\zeta}{z}$	$\mathsf{a} = 2 - \frac{2}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{2}{d_\phi+\zeta-z}$
LR	$\tau = 2 - \frac{1}{d+\zeta}$	$\tau_\phi = 2 - \frac{1}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-1+\zeta}{z}$	$\mathsf{a} = 2 - \frac{1}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{1}{d_\phi+\zeta-z}$

	d	ζ	z	τ	τ_ϕ	α	a	a_ϕ	γ
SR	1	1.25	1.433	1.11	0.4	1.17	−0.45	12.9	1.57
	2	0.75	1.56	1.27	−0.67	1.48	0.32	4.47	1.76
	3	0.34	1.74	1.40	−3.88	1.77	0.75	3.43	1.92
LR	1	0.39	0.74	1.28	−0.56	1.53	0.46	4.86	1.88

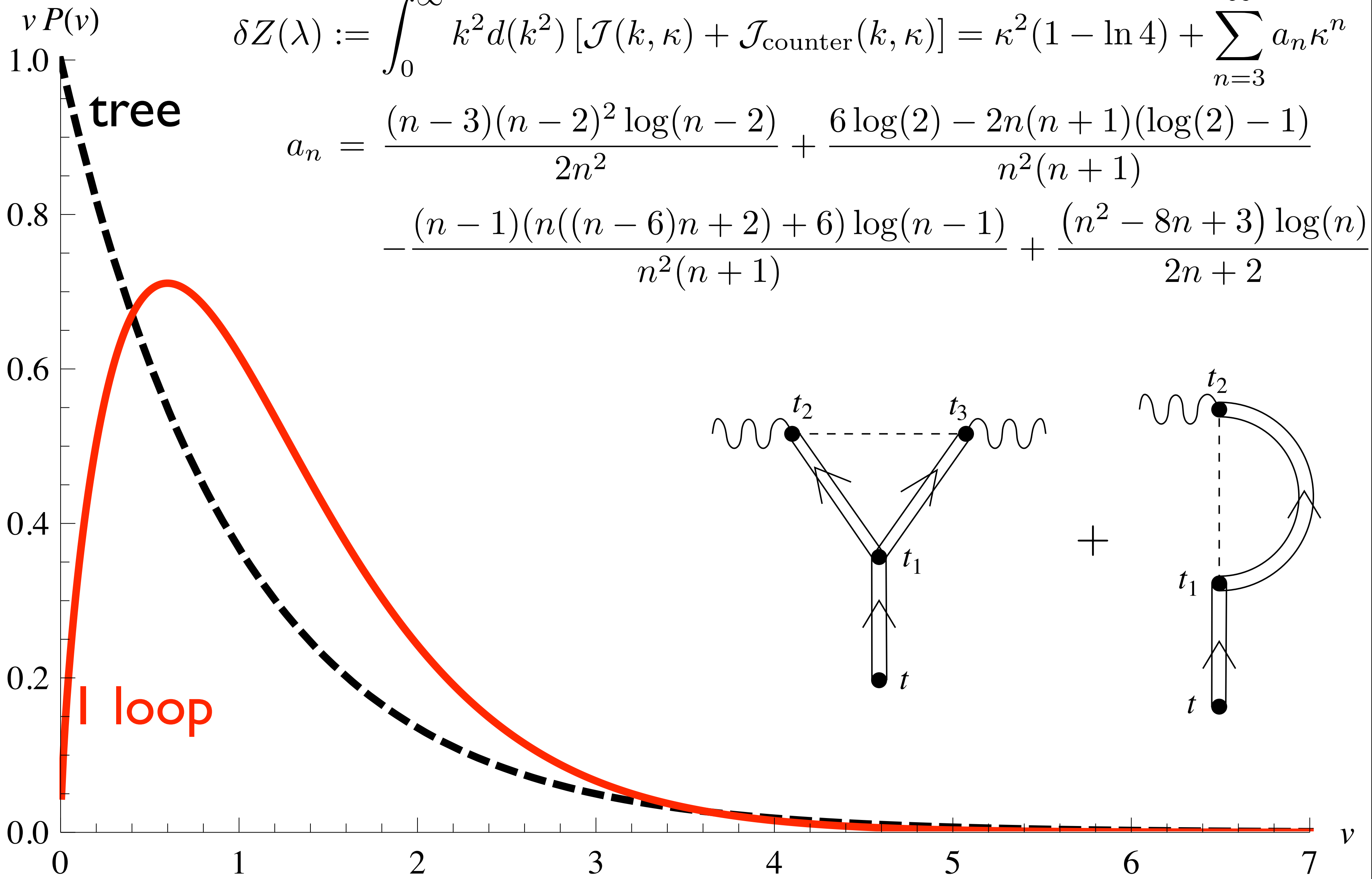
$$S \sim_{S \ll 1} T^{\gamma}$$

$$\gamma = \frac{d+\zeta}{z}$$

Velocity distribution in avalanche: tree + loops

$$\delta Z(\lambda) := \int_0^\infty k^2 d(k^2)\,[\mathcal{J}(k,\kappa) + \mathcal{J}_{\rm counter}(k,\kappa)] = \kappa^2(1-\ln 4) + \sum_{n=3}^{\infty} a_n \kappa^n$$

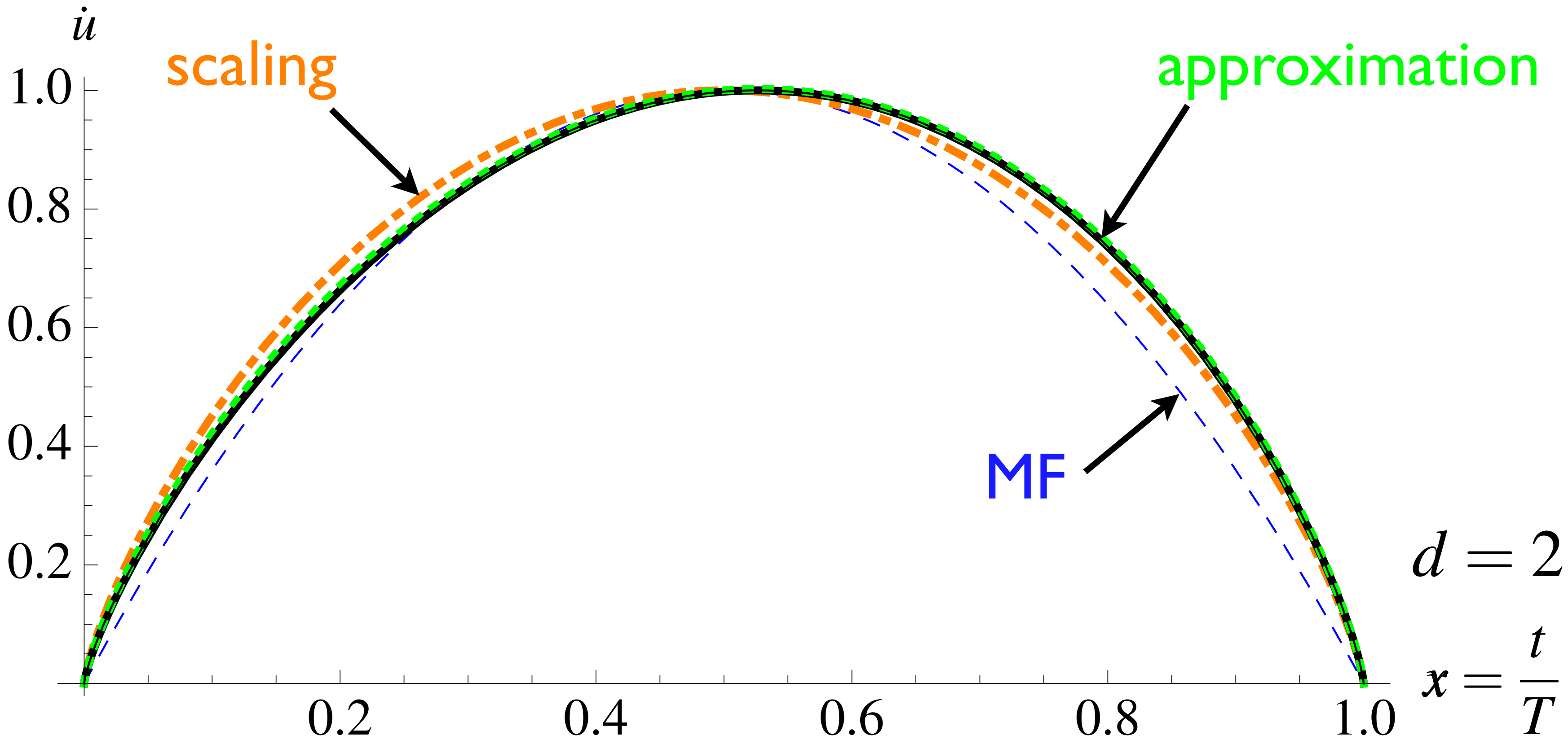
$$a_n = \frac{(n-3)(n-2)^2\log(n-2)}{2n^2} + \frac{6\log(2) - 2n(n+1)(\log(2)-1)}{n^2(n+1)} - \frac{(n-1)(n((n-6)n+2)+6)\log(n-1)}{n^2(n+1)} + \frac{\left(n^2-8n+3\right)\log(n)}{2n+2}$$



Shape at fixed duration

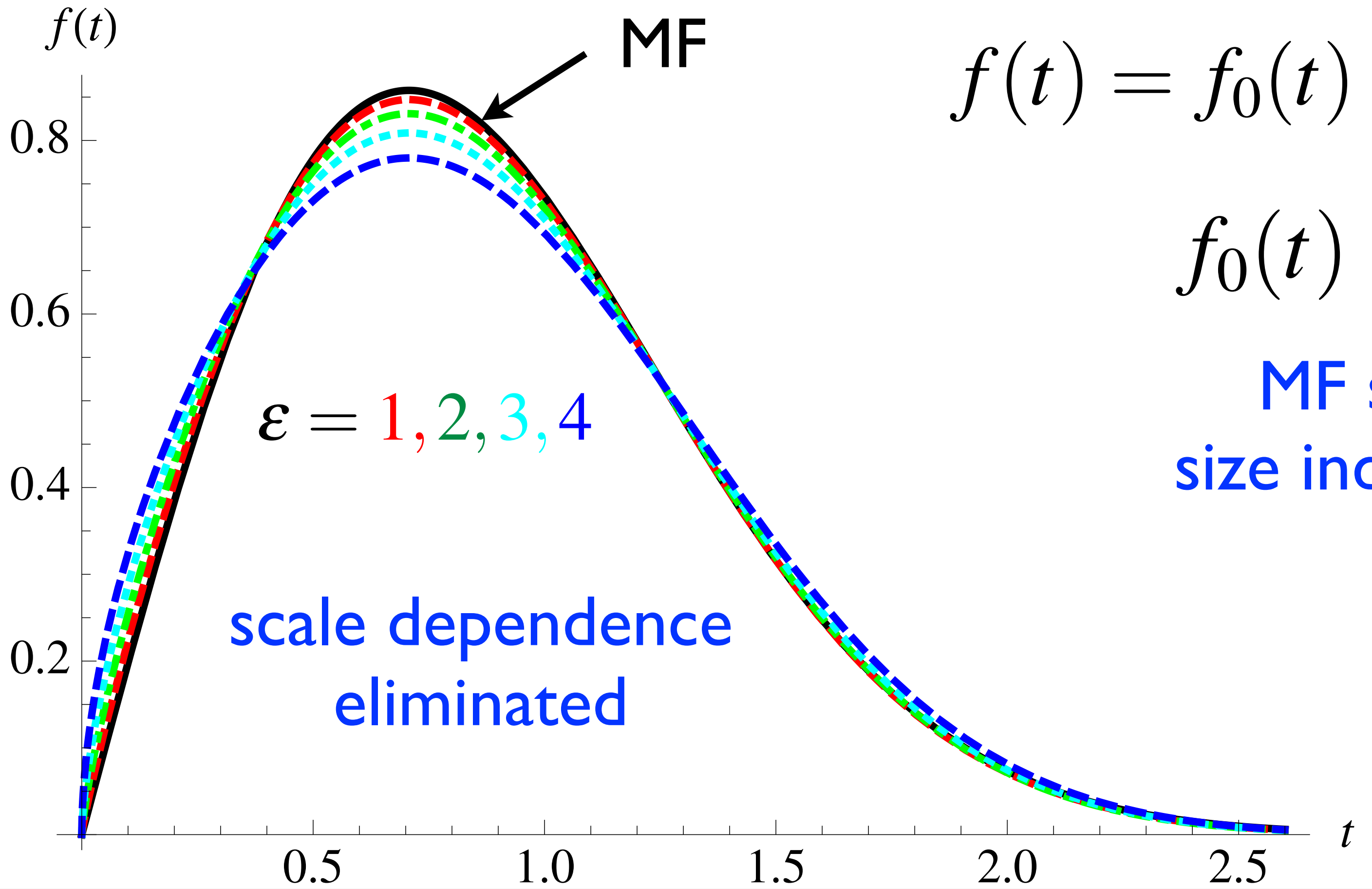
$$\left\langle \dot{u}\left(x=\frac{t}{T}\right)\right\rangle = \mathcal{N}\Big[Tx(1-x)\Big]^{1+\frac{2\alpha}{d_c}} \exp\left(\frac{8\alpha}{d_c}\left[\mathrm{Li}_2(1-x)-\mathrm{Li}_2\left(\frac{1-x}{2}\right)+\frac{x\log(2x)}{x-1}+\frac{(x+1)\log(x+1)}{2(1-x)}\right]\right)$$

$$\langle \dot{u}(x)\rangle \simeq \Big[Tx(1-x)\Big]^{\gamma-1}\exp\left(\mathcal{A}\left[\frac{1}{2}-x\right]\right) \qquad \mathcal{A}\approx -0.336\left(1-\frac{d}{d_c}\right)$$



Shape at fixed (small) size

$$\dot{u}(t,S) = S\left(\frac{S}{S_m}\right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m}\Big/\left(\frac{S}{S_m}\right)^{\frac{1}{\gamma}}\right)$$



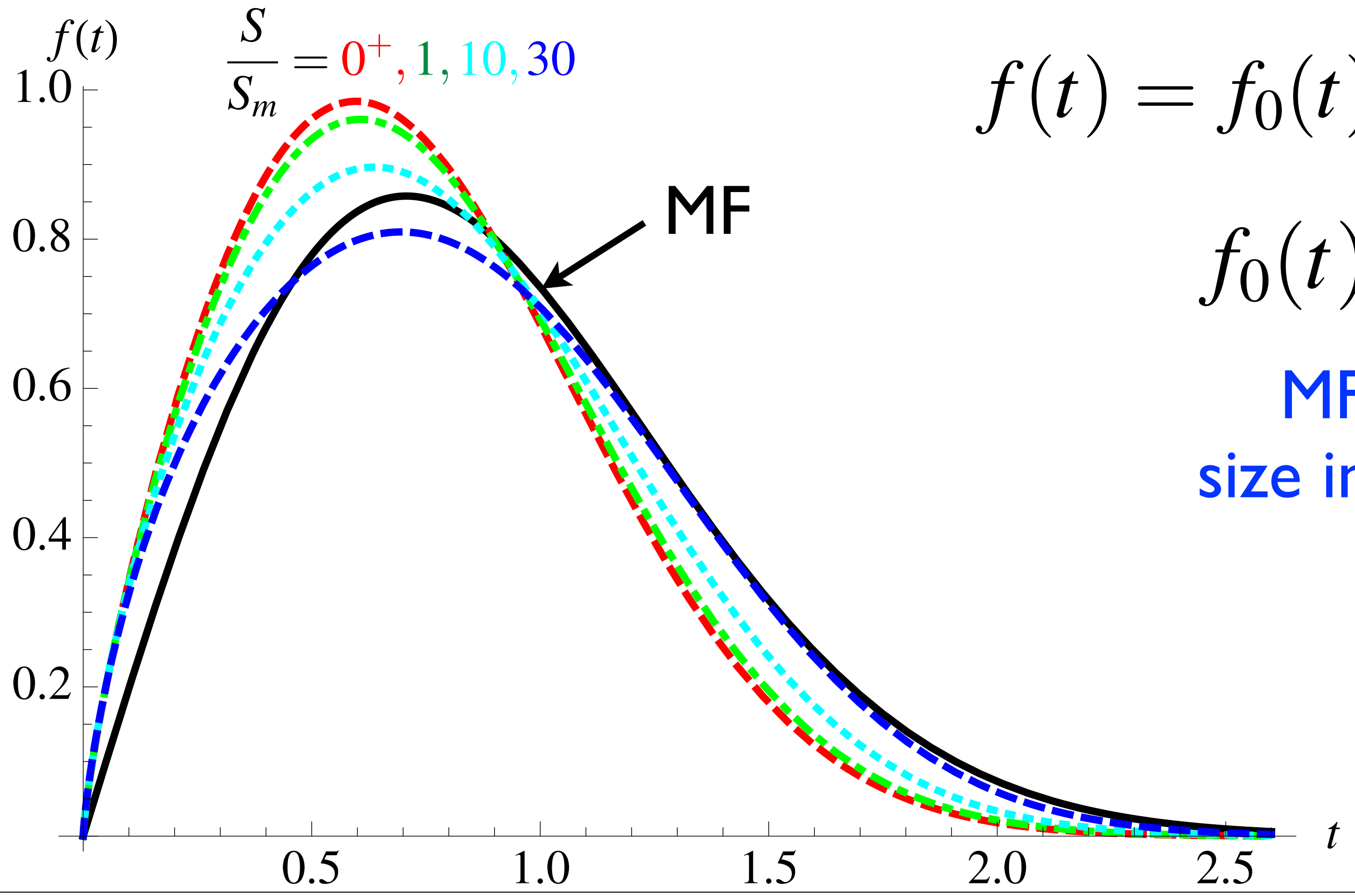
$$f(t) = f_0(t) + \frac{\alpha}{2}\delta f(t)$$

$$f_0(t) = 2te^{-t^2}$$

MF shape is
size independent!

Shape at fixed size: size dependence

$$\dot{u}(t,S) = S\left(\frac{S}{S_m}\right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \Big/ \left(\frac{S}{S_m}\right)^{\frac{1}{\gamma}}\right)$$



$$f(t) = f_0(t) + \frac{\alpha}{2}\delta f(t)$$

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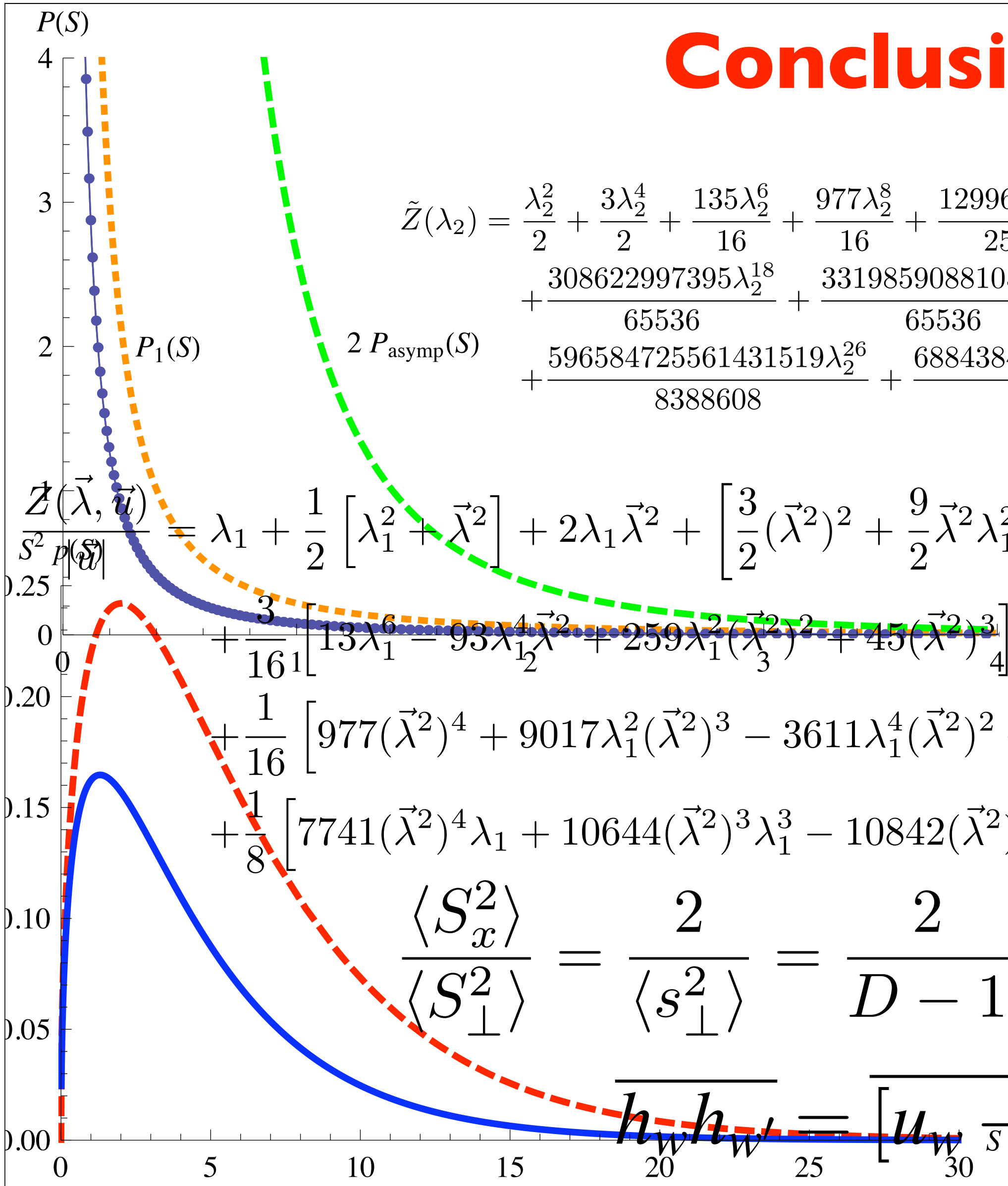
MF shape is size independent!

Conclusions



$$\tilde{Z}(\lambda_2) = \frac{\lambda_2^2}{2} + \frac{3\lambda_2^4}{2} + \frac{135\lambda_2^6}{16} + \frac{977\lambda_2^8}{16} + \frac{129963\lambda_2^{10}}{256} + \frac{1180377\lambda_2^{12}}{256} + \frac{91202891\lambda_2^{14}}{2048} + \frac{921684933\lambda_2^{16}}{2048} + \frac{308622997395\lambda_2^{18}}{65536} + \frac{3319859088105\lambda_2^{20}}{65536} + \frac{292183179645897\lambda_2^{22}}{524288} + \frac{3274855720092783\lambda_2^{24}}{524288} + \frac{5965847255614315 19\lambda_2^{26}}{8388608} + \frac{688438462700126 0037\lambda_2^{28}}{8388608} + \frac{642973844837708624403\lambda_2^{30}}{67108864} + O\left(\lambda_2^{31}\right)$$

$$\frac{Z(\vec{\lambda},\vec{u})}{|\vec{u}|} = \lambda_1 + \frac{1}{2}\left[\lambda_1^2 + \vec{\lambda}^2\right] + 2\lambda_1\vec{\lambda}^2 + \left[\frac{3}{2}(\vec{\lambda}^2)^2 + \frac{9}{2}\vec{\lambda}^2\lambda_1^2 - \lambda_1^4\right] + \left[-\frac{3}{2}\lambda_1^5 + 3\lambda_1^3\vec{\lambda}^2 + \frac{25}{2}\lambda_1(\vec{\lambda}^2)^2\right]$$
$$+\frac{3}{16}\left[13\lambda_1^6 - 93\lambda_1^4\vec{\lambda}^2 + 259\lambda_1^2(\vec{\lambda}^2)^2 + 45(\vec{\lambda}^2)^3\right] + \left[14\lambda_1^7 - 57\lambda_1^5\vec{\lambda}^2 + 72\lambda_1^3(\vec{\lambda}^2)^2 + 103\lambda_1(\vec{\lambda}^2)^3\right]$$
$$+\frac{1}{16}\left[977(\vec{\lambda}^2)^4 + 9017\lambda_1^2(\vec{\lambda}^2)^3 - 3611\lambda_1^4(\vec{\lambda}^2)^2 + 287\lambda_1^6\vec{\lambda}^2 + 194\lambda_1^8\right]$$
$$+\frac{1}{8}\left[7741(\vec{\lambda}^2)^4\lambda_1 + 10644(\vec{\lambda}^2)^3\lambda_1^3 - 10842(\vec{\lambda}^2)^2\lambda_1^5 + 4548(\vec{\lambda}^2)\lambda_1^7 - 651\lambda_1^9\right] + O(\lambda^9)$$

$$\frac{\langle S_x^2\rangle}{\langle S_\perp^2\rangle} = \frac{2}{\langle s_\perp^2\rangle} = \frac{2}{D-1} \qquad S_m := \frac{\langle S^2\rangle}{2\langle S\rangle} = \frac{|\Delta'(0^+)|}{m^4}$$

$$\overline{h_w h_{w'}} = \overline{[u_w - w][u_{w'} - w']} = \frac{\Delta(w-w')}{L^d m^4}$$

??? WHERE ARE THE EXPERIMENTS ???