Avalanche Statistics in Disordered Visco-Elastic Interfaces

François Landes

LPTMS, Orsay

Driven Disordered Systems 2014 - 2014/06/06

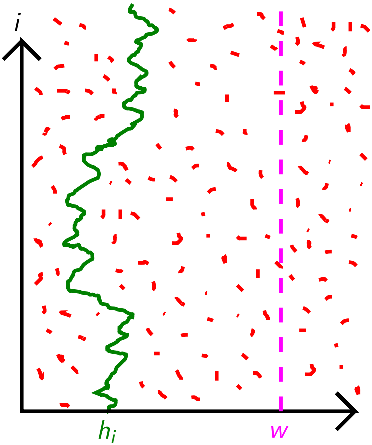
June 6, 2014

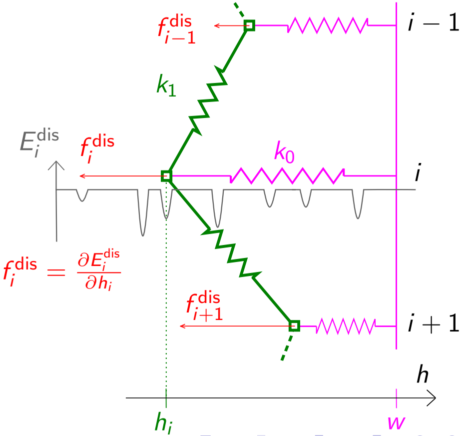
In collaboration with:
E.A. Jagla (Bariloche, Argentina),
Alberto Rosso (Orsay, France).

Phys. Rev. Lett. **112**, 174301

(Conventional) Depinning of an interface (1)

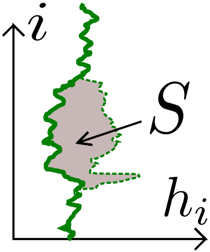
- Out-of-equilibrium: slow Driving $w(t) = V_0 t$
- Disorder: random distribution $f_i^{\text{dis}}(h_i)$
- Competition: Disorder VS Elasticity
  $\eta \partial_t h_i = k_0(w - h_i) + f_i^{\text{dis}}(h_i) + k_1 \nabla^2 h_i$
  Two time scales, $\eta \ll dh/V_0$

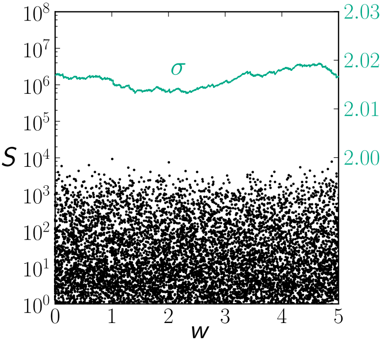


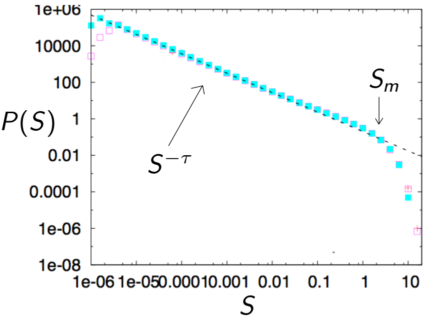


(Conventional) Depinning of an interface (2)

- ⇒ scale-free distribution of avalanches
  $P(S) \sim S^{-\tau} e^{-S/S_m}$
- ⇒ no correlations
- ⇒ Stationary stress $\sigma \equiv \overline{k_0(w-h)}$





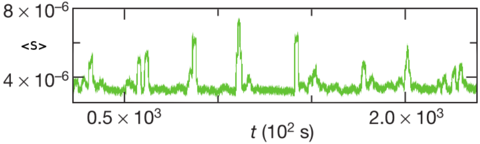


Works for: cracks propagation, magnetization domains, etc.

Correlations?

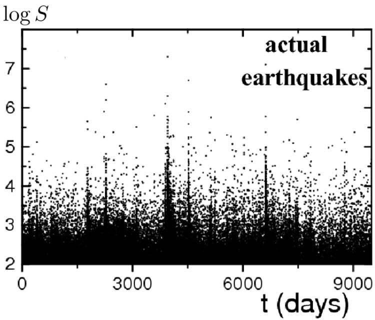
- compression of solids, earthquakes dynamics: correlations !

low strain rate: $\gamma = 10^{-6}\ s^{-1}$



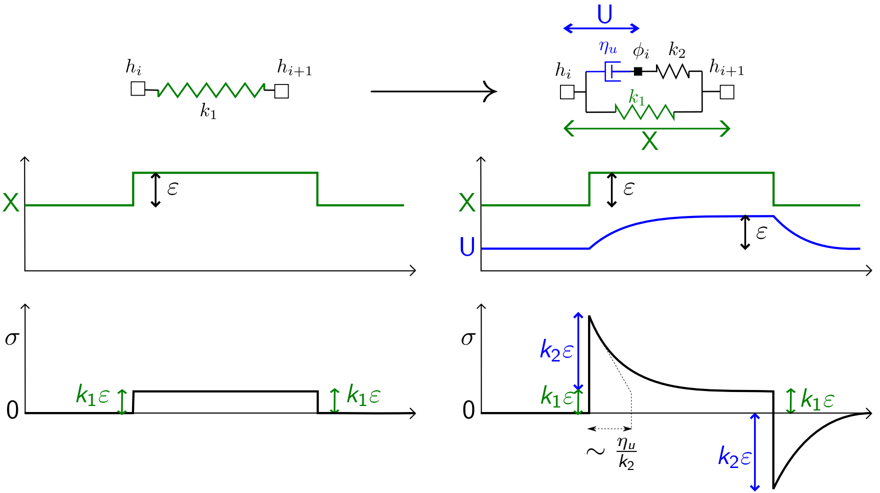
Micro-crystals compression

(S. Papanikolaou *et.al.*, Nature 490, 51721 (2012)).



- How to get correlations in Depinning Framework ?

Simple Model of Viscoelastic Material

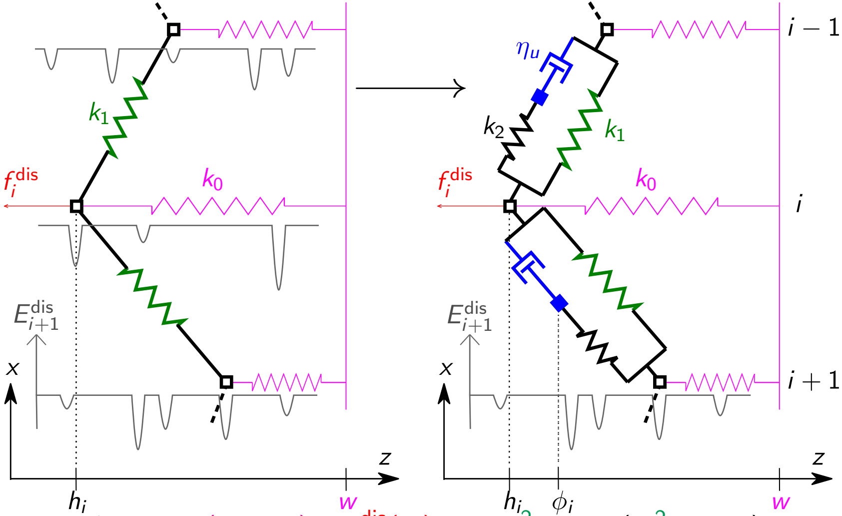


Purely elastic $\longrightarrow$ **visco** - elastic

$F_{h_i \to h_{i+1}} = k(h_{i+1} - h_i) \longrightarrow F_{\phi \to h} = \eta_u \partial_t (h - \phi) + \textit{elastic int.}$

One degree of freedom h_i per site $\longrightarrow$ Two degrees of freedom h_i, ϕ_i per site

Visco-Elastic Depinning: Definition



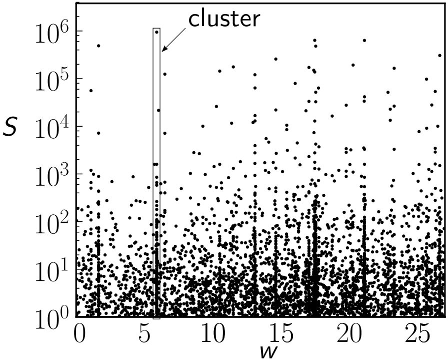
$$\eta\partial_t h_i = k_0(w - h_i) + f_i^{\mathrm{dis}}(h_i) + k_1\nabla^2 h_i + k_2(\nabla^2 h_i - u_i)$$

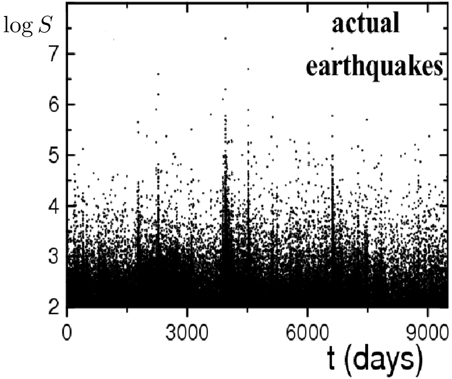
$$\eta_u\partial_t u_i = k_2(\nabla^2 h_i - u_i),$$

with $u_i \equiv \phi_i - h_i + h_{i-1} - \phi_{i-1}$

Numerical Results, **Two Dimensions**

Visco-elastic model: Aftershocks!

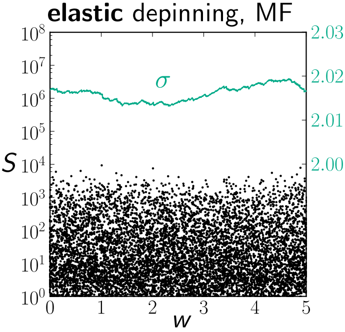


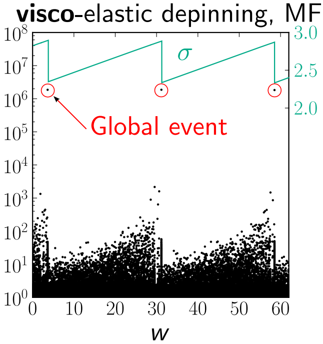


Also:

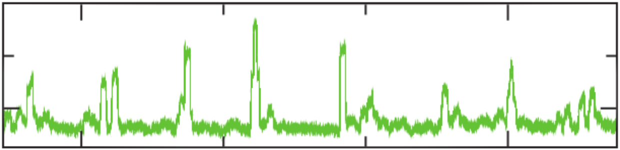
Avalanche distribution: $P(S) \sim S^{-\tau}, \tau \in [1.7, 1.8]$, realistic for Earthquakes.

Numerical Results, **Mean Field**





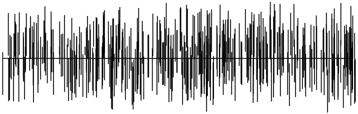
- Strong correlations appear: aftershocks, etc.
- Global avalanches, periodically.



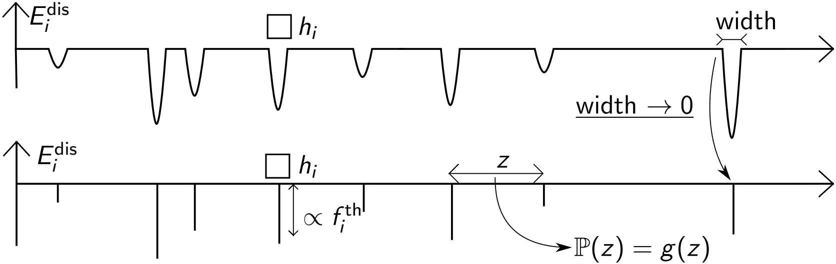
(S. Papanikolaou *et.al.*, Nature 490, 51721 (2012)).

Approximation: Identical Wells

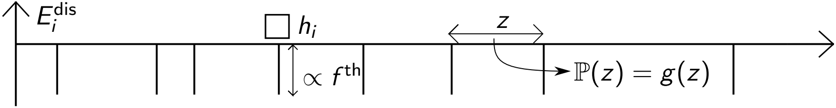
- White noise:



- Simplify: narrow wells (D.S. Fisher, Physics Reports 301, 113–150 (1998))



- Simplify more: same depth for all wells, $f_i^{\text{th}}(h_i) \equiv f^{\text{th}} \equiv \text{const.}$



Mean Field: conventional Depinning

$\delta_i \equiv$ how much force before getting out of pinning well

$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$

$P_w(\delta) \to P_{w+\mathrm{d}w}(\delta) =\ ?$
(0) Driving: shift in δ by $k_0 \mathrm{d}w$: $P(\delta) \longleftarrow P(\delta + k_0 \mathrm{d}w)$

(1) fraction $P(0)k_0\mathrm{d}w$ of system jumps from 0 to $\delta = z(k_0 + k_1)$:
$P(\delta)\mathrm{d}\delta \longleftarrow P(\delta)\mathrm{d}\delta + P(0)k_0\mathrm{d}w.g(z)\mathrm{d}z$

(0) $\oplus$ (1) : $\frac{P_{\text{step1}}(\delta) - P_w(\delta)}{k_0 \mathrm{d}w} = \frac{\partial P_w}{\partial \delta}(\delta) + P_w(0)\frac{g\left(\frac{\delta}{k_0+k_1}\right)}{k_0+k_1}$

(2) jumps of $z \Rightarrow$ increase in $\overline{h}$: shift in δ by $P(0)\overline{z}k_1.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: shift in δ by $(P(0)\overline{z}k_1)^2.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: $\quad \ldots$

$\ldots$ until shift ≈ 0.

Mean Field: conventional Depinning

$\delta_i \equiv$ how much force before getting out of pinning well

$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$

$P_w(\delta) \to P_{w+\mathrm{d}w}(\delta) = \ ?$
(0) Driving: shift in δ by $k_0 \mathrm{d}w$: $P(\delta) \longleftarrow P(\delta + k_0 \mathrm{d}w)$

(1) fraction $P(0)k_0\mathrm{d}w$ of system jumps from 0 to $\delta = z(k_0 + k_1)$:
$P(\delta)\mathrm{d}\delta \longleftarrow P(\delta)\mathrm{d}\delta + P(0)k_0\mathrm{d}w.g(z)\mathrm{d}z$

(0) $\oplus$ (1) : $\frac{P_{\text{step1}}(\delta) - P_w(\delta)}{k_0 \mathrm{d}w} = \frac{\partial P_w}{\partial \delta}(\delta) + P_w(0)\frac{g\left(\frac{\delta}{k_0+k_1}\right)}{k_0+k_1}$

(2) jumps of $z \Rightarrow$ increase in $\overline{h}$: shift in δ by $P(0)\overline{z}k_1.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: shift in δ by $(P(0)\overline{z}k_1)^2.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: $\quad \ldots$

$\ldots$ until shift ≈ 0.

Mean Field: conventional Depinning

$\delta_i \equiv$ how much force before getting out of pinning well

$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$

$P_w(\delta) \to P_{w+\mathrm{d}w}(\delta) = \ ?$
(0) Driving: shift in δ by $k_0 \mathrm{d}w$: $P(\delta) \longleftarrow P(\delta + k_0 \mathrm{d}w)$

(1) fraction $P(0)k_0\mathrm{d}w$ of system jumps from 0 to $\delta = z(k_0 + k_1)$:
$P(\delta)\mathrm{d}\delta \longleftarrow P(\delta)\mathrm{d}\delta + P(0)k_0\mathrm{d}w.g(z)\mathrm{d}z$

(0) $\oplus$ (1) : $\frac{P_{\mathrm{step1}}(\delta) - P_w(\delta)}{k_0 \mathrm{d}w} = \frac{\partial P_w}{\partial \delta}(\delta) + P_w(0) \frac{g\left(\frac{\delta}{k_0+k_1}\right)}{k_0+k_1}$

(2) jumps of $z \Rightarrow$ increase in $\overline{h}$: shift in δ by $P(0)\overline{z}k_1.k_0\mathrm{d}w$
$\to$ more steps $(0), (1) \to (2)$: shift in δ by $(P(0)\overline{z}k_1)^2.k_0\mathrm{d}w$
$\to$ more steps $(0), (1) \to (2)$: $\ldots$

$\ldots$ until shift ≈ 0.

Mean Field: conventional Depinning

$\delta_i \equiv$ how much force before getting out of pinning well

$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$

$P_w(\delta) \to P_{w+\mathrm{d}w}(\delta) =\ ?$
(0) Driving: shift in δ by $k_0\mathrm{d}w$: $P(\delta) \longleftarrow P(\delta + k_0\mathrm{d}w)$

(1) fraction $P(0)k_0\mathrm{d}w$ of system jumps from 0 to $\delta = z(k_0 + k_1)$:
$P(\delta)\mathrm{d}\delta \longleftarrow P(\delta)\mathrm{d}\delta + P(0)k_0\mathrm{d}w.g(z)\mathrm{d}z$

(0) $\oplus$ (1) : $\frac{P_{\mathrm{step1}}(\delta)-P_w(\delta)}{k_0\mathrm{d}w} = \frac{\partial P_w}{\partial\delta}(\delta) + P_w(0)\frac{g\left(\frac{\delta}{k_0+k_1}\right)}{k_0+k_1}$

(2) jumps of $z \Rightarrow$ increase in $\overline{h}$: shift in δ by $P(0)\overline{z}k_1.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: shift in δ by $(P(0)\overline{z}k_1)^2.k_0\mathrm{d}w$
$\to$ more steps $(0),(1) \to (2)$: $\quad\dots$

$\dots$ until shift ≈ 0.

Mean Field: conventional Depinning

$\delta_i \equiv$ how much force before getting out of pinning well

$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$

$P_w(\delta) \to P_{w+\mathrm{d}w}(\delta) = \ ?$
(0) Driving: shift in δ by $k_0 \mathrm{d}w$: $P(\delta) \longleftarrow P(\delta + k_0 \mathrm{d}w)$

(1) fraction $P(0)k_0\mathrm{d}w$ of system jumps from 0 to $\delta = z(k_0 + k_1)$:
$P(\delta)\mathrm{d}\delta \longleftarrow P(\delta)\mathrm{d}\delta + P(0)k_0\mathrm{d}w.g(z)\mathrm{d}z$

(0) $\oplus$ (1) : $\frac{P_{\mathrm{step1}}(\delta) - P_w(\delta)}{k_0 \mathrm{d}w} = \frac{\partial P_w}{\partial \delta}(\delta) + P_w(0)\frac{g\left(\frac{\delta}{k_0+k_1}\right)}{k_0+k_1}$

(2) jumps of $z \Rightarrow$ increase in $\overline{h}$: shift in δ by $P(0)\overline{z}k_1.k_0\mathrm{d}w$
$\to$ more steps $(0), (1) \to (2)$: shift in δ by $(P(0)\overline{z}k_1)^2.k_0\mathrm{d}w$
$\to$ more steps $(0), (1) \to (2)$: $\quad \ldots$

... until shift ≈ 0.

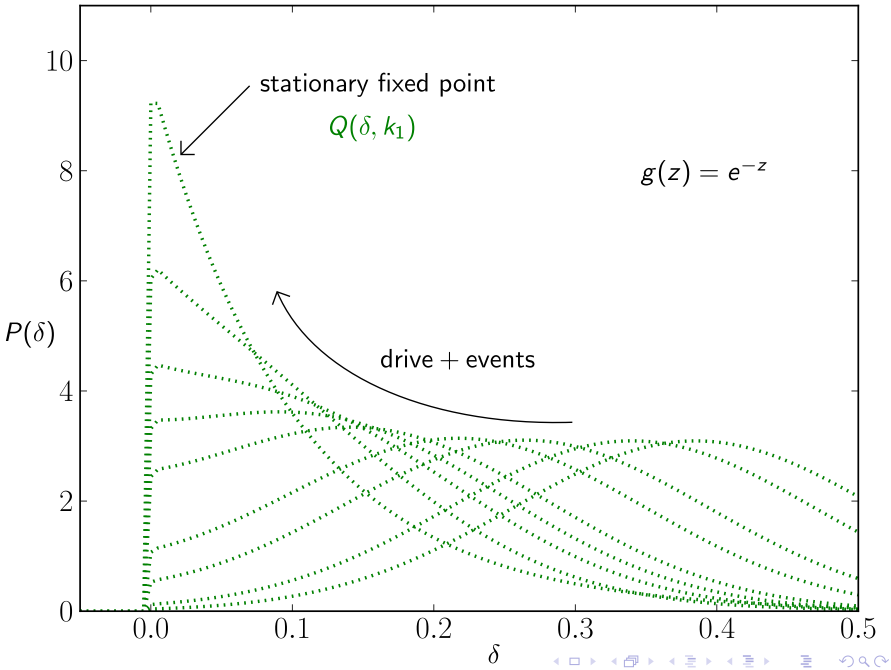
Mean Field Solution

The stationary regime (fixed point) fulfils:

$$0 = \frac{\partial P}{\partial \delta}(\delta) + P(0)\frac{g(\frac{\delta}{k_0+k_1})}{k_0 + k_1} \tag{1}$$

Solution:

$$P(\delta) = \frac{1 - \int_0^{\frac{\delta}{k_0+k_1}} g(z)\mathrm{d}z}{\overline{z}(k_0 + k_1)} \equiv Q(\delta, k_1) \tag{2}$$


stationary fixed point
$Q(\delta, k_1)$
$g(z) = e^{-z}$
drive + events
$P(\delta)$
δ
0
2
4
6
8
10
0.0
0.1
0.2
0.3
0.4
0.5

Mean Field Solution

From any initial state, fixed point reached in finite time.

If $P(0) < \bar{z}k_1$: finite avalanches with cutoff
$S_m = (1 - P(0)\bar{z}k_1)^{-2} = \left(\frac{k_0+k_1}{k_0}\right)^2$

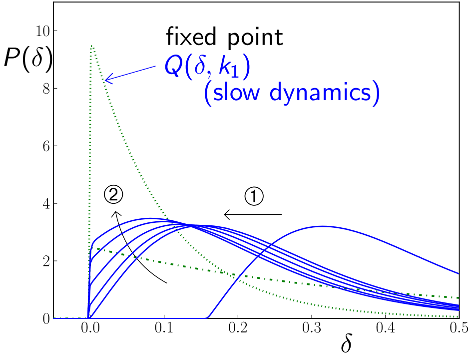
If $P(0) \geq \bar{z}k_1$: divergent avalanches, more analysis needed.

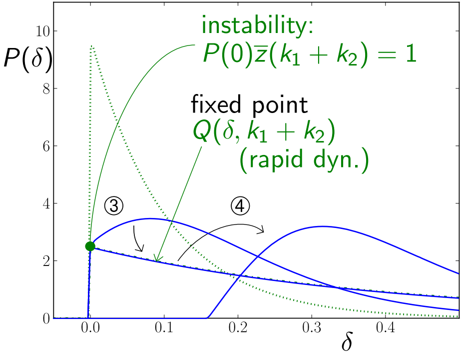
Exact results: Mean Field (1)

Two limiting regimes compete ⇒ cycle emerges

$$\delta_i = f^{\text{th}} - k_0(w - h_i) - k_1(\bar{h} - h_i) - k_2(\bar{h} - h_i - u_i) \quad (3)$$

$$\eta_u \partial_t u_i = k_2(\bar{h} - h_i - u_i) \quad (4)$$

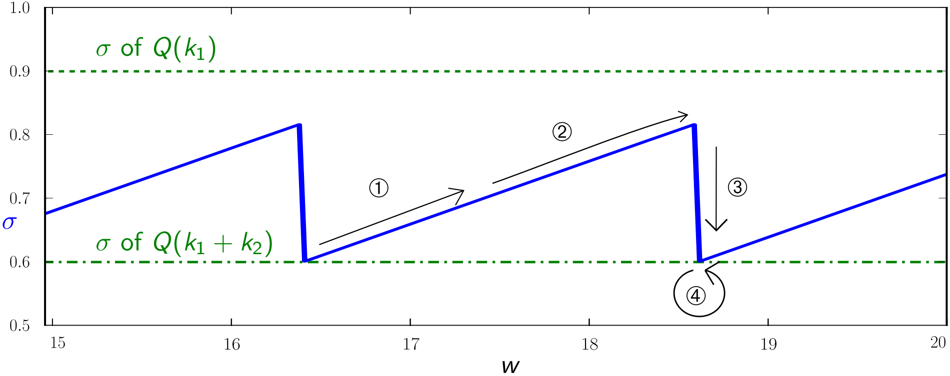




$k_0 = 0.001, k_1 = 0.1, k_2 = 0.3$

Exact results: Mean Field (2)

Stress oscillates periodically:



Conclusions

- understand aftershocks or avalanches' correlations: **visco-elasticity** is a good candidate
- **microscopic** time $\eta_u \Rightarrow$ **emerging time scale** (periodicity)
  (Non-Equilibrium Non-Stationary State)
- **Fokker-Planck** analysis allows to understand the **time-dependent evolution** of the system state.
- periodic behaviour explained as a competition between (fast-dynamics) **stable** depinning critical point $Q(\delta, k_1 + k_2)$ VS **unstable** one (but attractive for the slow dynamics) $Q(\delta, k_1)$

Thank You!

Three characteristic times

Three time scales: internal ava. time , inter-aftershock , driving

$\eta \ll \eta_u \ll \mathrm{d}h/V_0$

$$\eta \partial_t h_i = k_0(w - h_i) + f_i^{\text{dis}}(h_i) + k_1 \nabla^2 h_i + k_2(\nabla^2 h_i - u_i) \quad (5)$$

$$\eta_u \partial_t u_i = k_2(\nabla^2 h_i - u_i), \quad (6)$$

- (i) During avalanches (time $\sim \eta$)
  $\Rightarrow$ dashpots are blocked, $u_i \approx$ const. interface h jumps
  $\rightarrow$ classical depinning *evolution*, with $k_1^{\text{eff}} = k_1 + k_2$
- (ii) Between avalanches (time $\sim \eta_u$)
  when h_i are pinned : relaxation of u_i: $u_i \rightarrow \nabla^2 h_i$.
  $\Rightarrow$ triggers new avalanches ($\simeq$ aftershocks)
  $\rightarrow$ drive towards classical depinning *state*, $k_1^{\text{eff}} = k_1$
- (iii) When all h_i pinned and all u_i relaxed (time $\sim \mathrm{d}h/V_0$):
  $\Rightarrow$ drive , $w \rightarrow w + \mathrm{d}w$

Mean Field: conventional Depinning (1) : Summary

- Fully-connected model: $(\nabla^2 h)_i \longrightarrow \overline{h} - h_i$
- equal pinning wells: $f_i^{\text{dis}}(h_i) \longrightarrow f^{\text{th}}, \mathbb{P}(z) = g(z)$
- Interface continuous motion discretized:
  $\{\partial_t h > 0 \Rightarrow \text{jump}\} \quad \longrightarrow \quad \{\delta_i < 0 \Rightarrow \text{jump}\}$
  with $\delta_i \equiv f^{\text{th}} - k_0(w - h_i) - k_1(\overline{h} - h_i)$
- All sites equivalent: Fokker-Planck analysis
  $\{h_i, \forall i\} \rightarrow \{\delta_i, \forall i\} \rightarrow P(\delta)$ describes whole system
  N blocks $\rightarrow$ N blocks $\rightarrow \infty$ blocks
- $\partial_t h = \cdots \rightarrow \partial_t P(\delta) =??$

Numerical Results: 2D

- locally pseudo periodic
- local stress:
  oscillations between two values