UMR Gulliver

Effets Collectifs & Matière Molle

# Jamming at finite T, a granular media experiment

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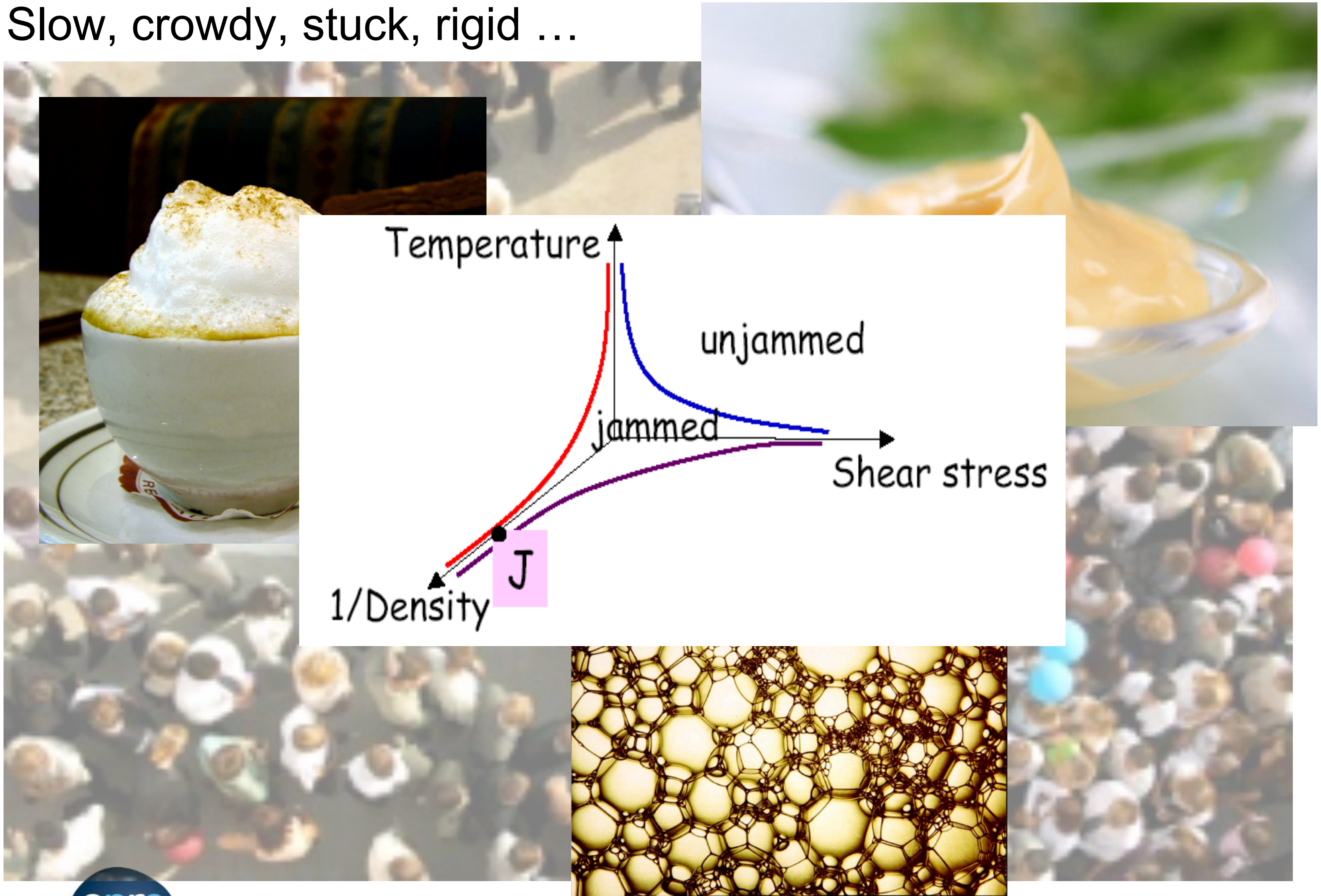
# Overview

- Introduction
  - Why studying Jamming in vibrated grains ?
- Dynamical signature of jamming in a system of brass, (hard) discs
  - Dynamical heterogeneities at minute scales
  - Open Issues
- Jamming in a system of photo-elastic (soft) discs
  - Role of the dynamics at the contact
  - Exploring the vicinity of point J
- Mechanical response to a point like disturbances
  - Journey of an intruder
  - Around an inflater

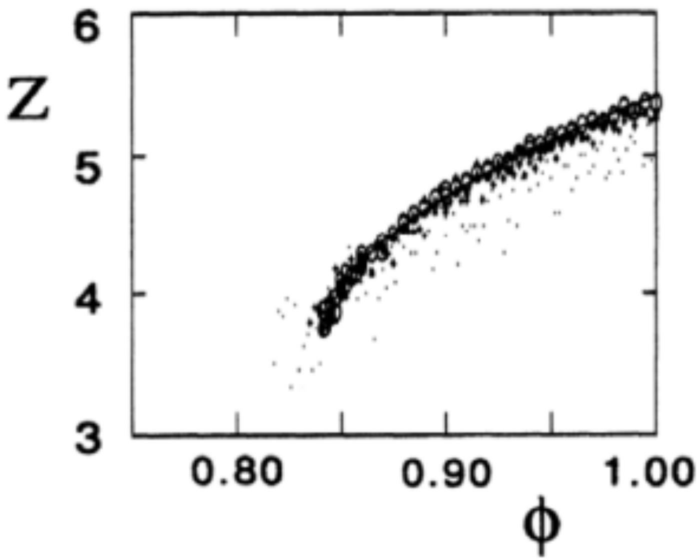
# Jamming in a very loose sense

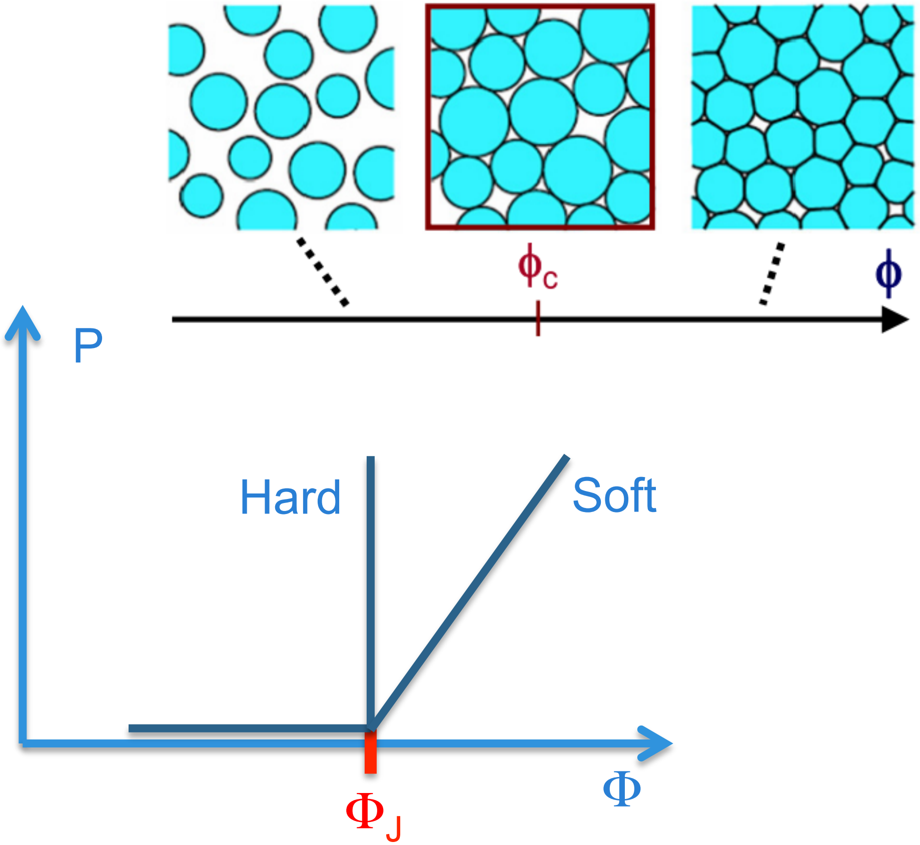
- Slow, crowdy, stuck, rigid …



# A paradigm: Jamming of soft spheres at T=0

- A well defined concept,
  (*O'Hern et al. (2002)* )




A geometrical transition

# of contacts jumps to $z_J = z_{iso}$

$\delta z = z - z_J \sim (\Phi-\Phi_J)^{1/2}$

g(r=d) = delta function

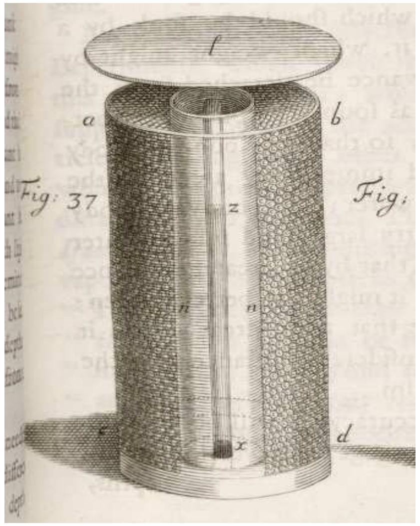
A mechanical transition

Pressure scales as prescribed by elasticity

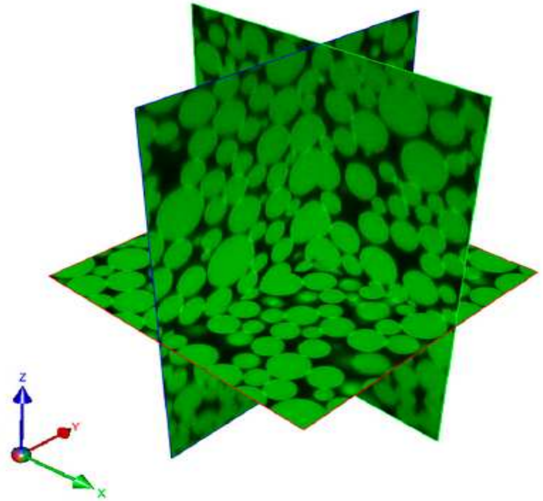
Elastic moduli scaling $K/k \sim \delta z^0$; $G/k \sim \delta z$

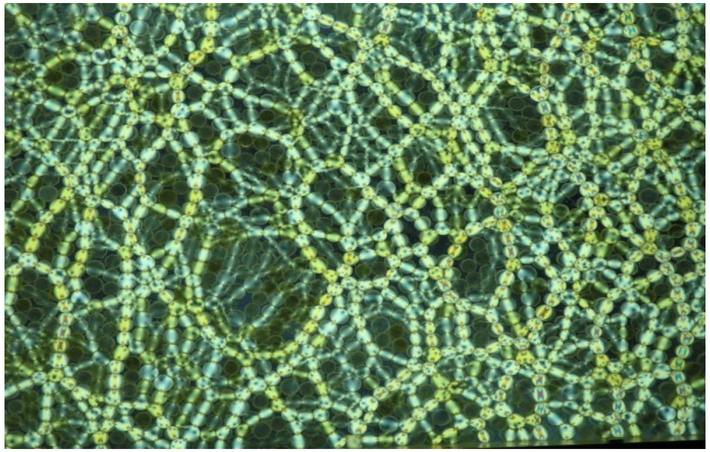
# Experimental realizations

*Green peas, Hales, 1727*

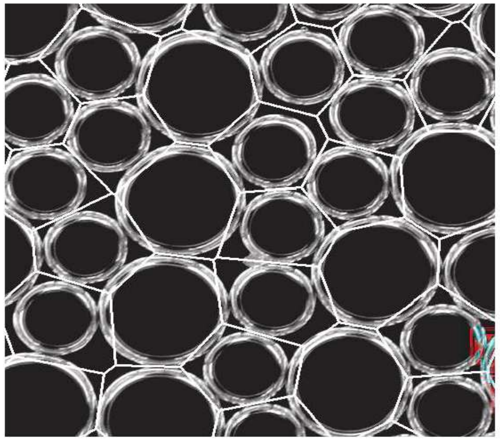



*Emulsion, Jorjadze et al., 2011*



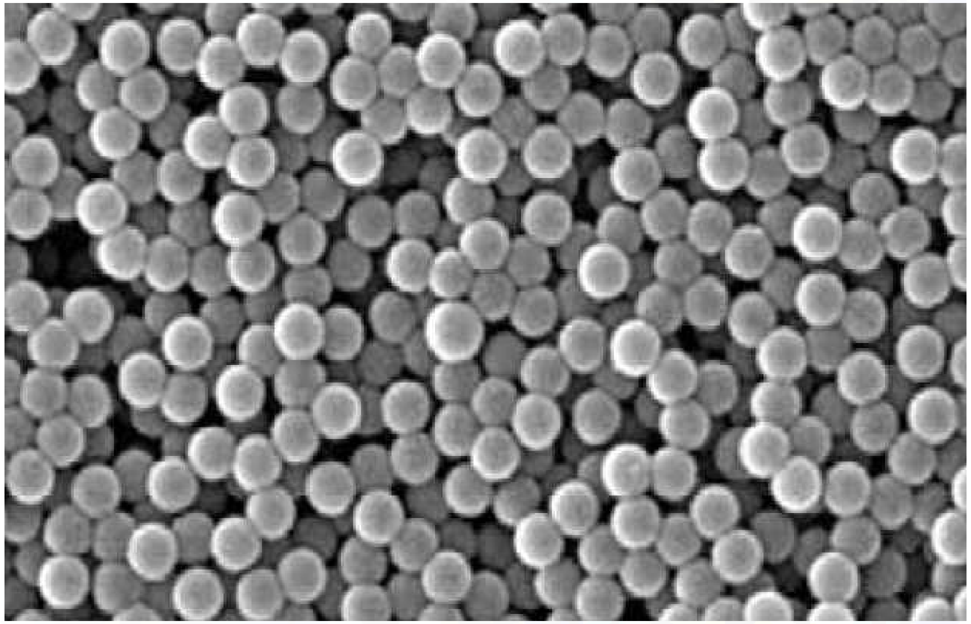


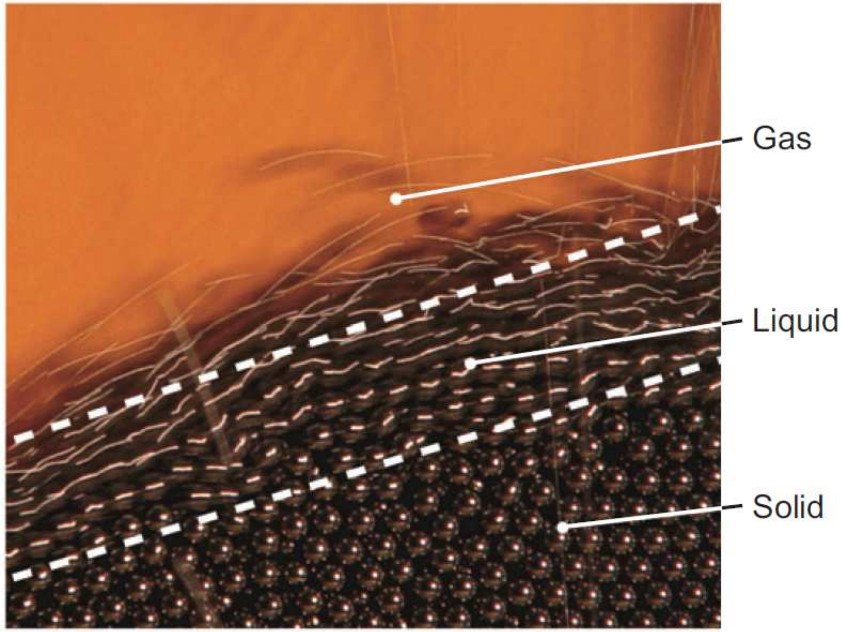
*Grains, Behringer*



*Foam, Katgert et van Hecke, 2010*

# What about *these* situations?





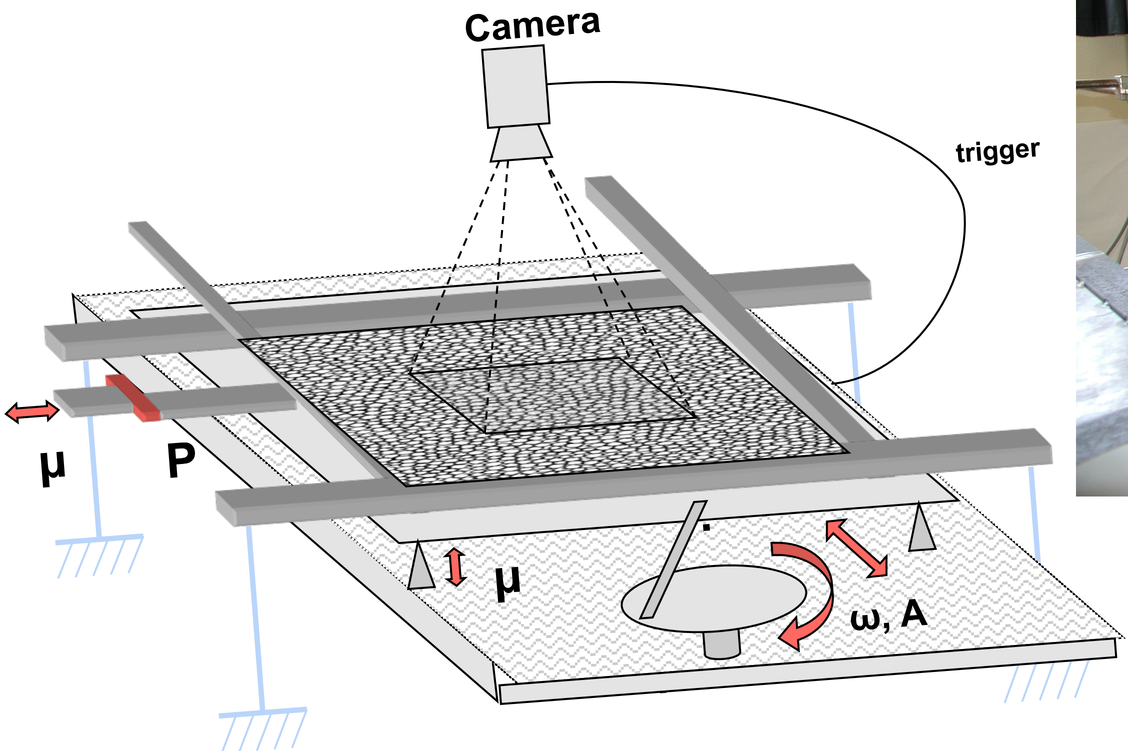

Colloidal suspensions
=> thermal agitation

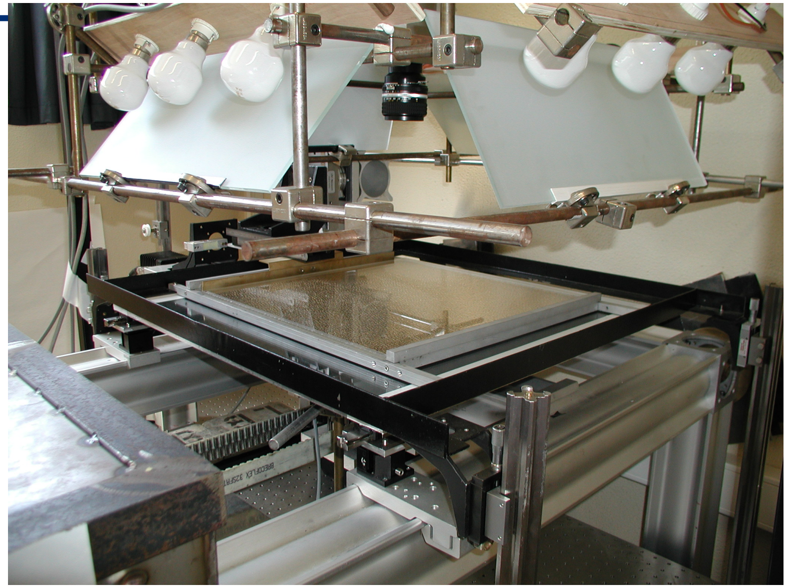
Dense granular flows
=> mechanical excitation

Control of Dynamics by Jamming scalings?

Effect of Dynamics on jammed systems?

# Jamming in a system of vibrated brass discs



- Horizontal vibration (ω=10 Hz, a=1cm)
- Bi-disperse : $d_s$ = 4mm $d_l$ = 5mm
- 8000 brass discs in the system (1500 tracked)
- Vibration-trigged camera
- Tunable volume
- Pressure measured on the side

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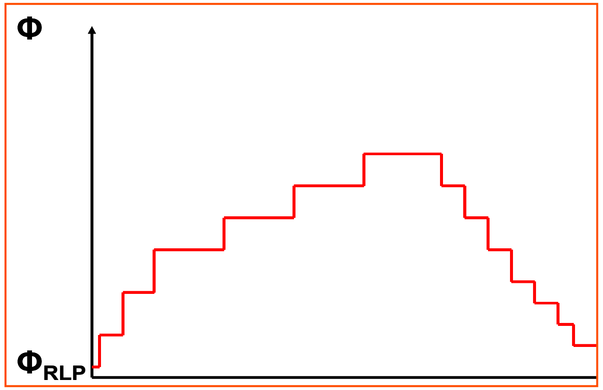
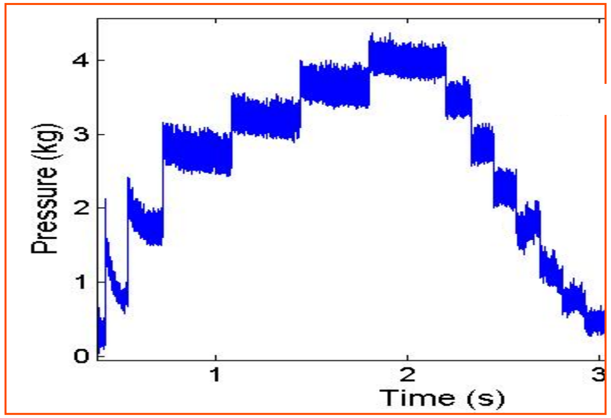
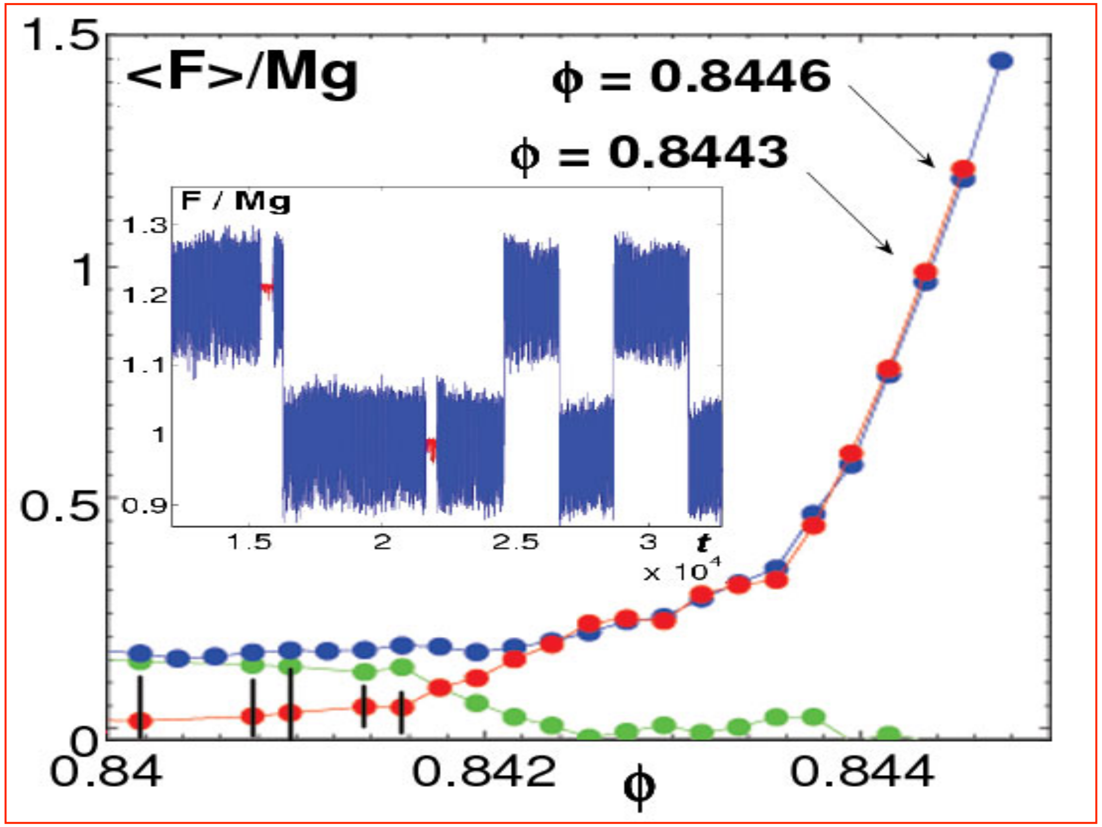
# Experimental protocol

- Increase packing fraction stepwise:
  - Allow for the slow relaxation of pressure
- Then decrease packing fraction and record dynamics



# Experimental protocol

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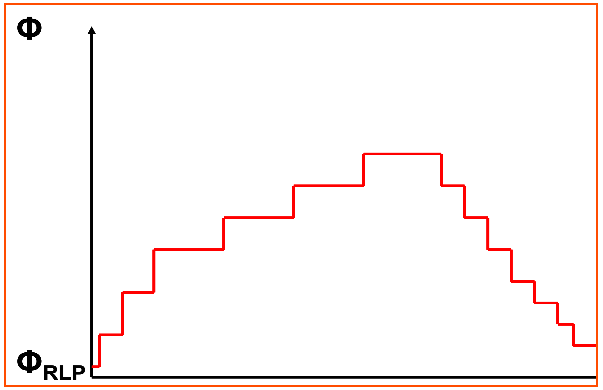
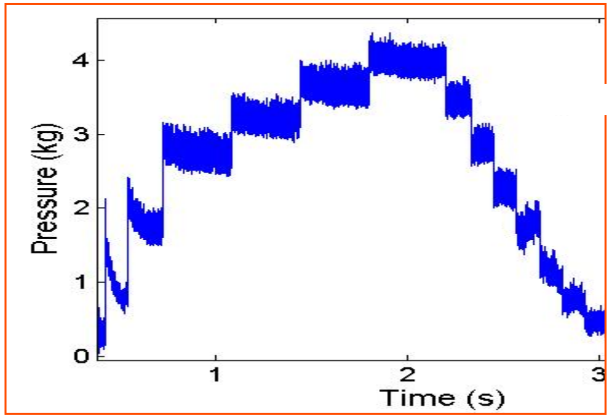
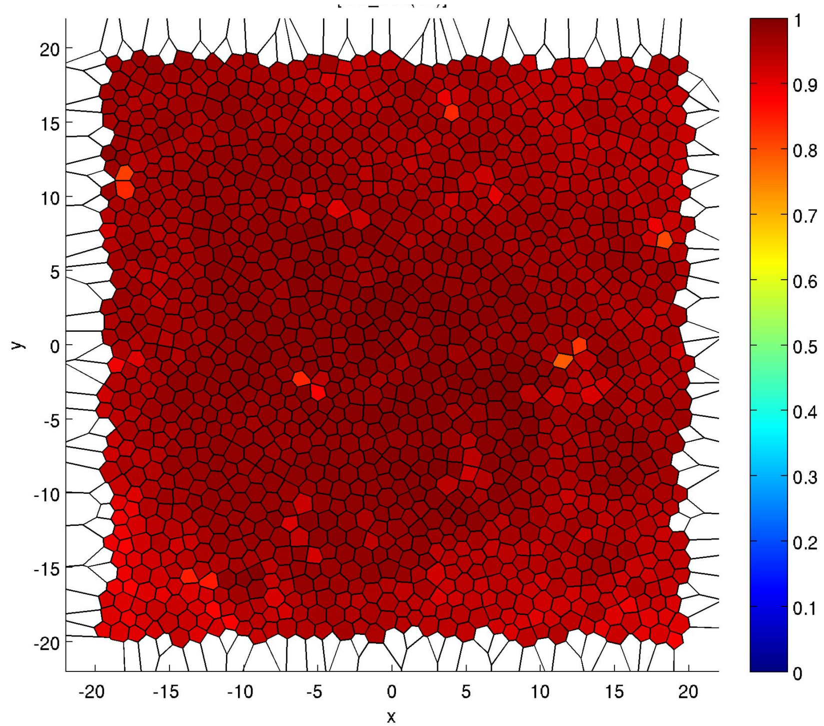


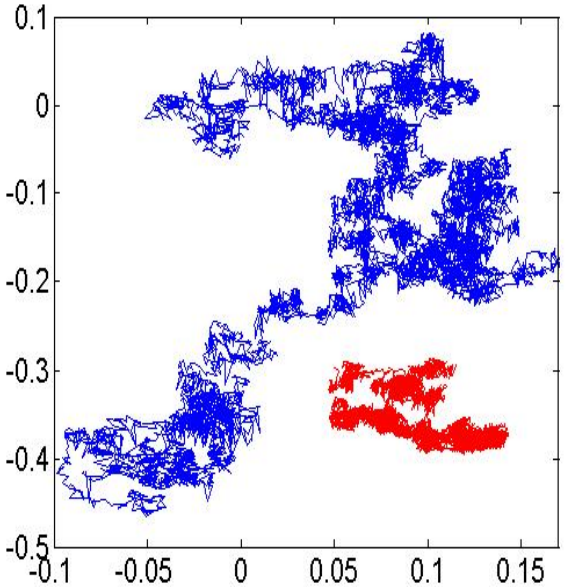



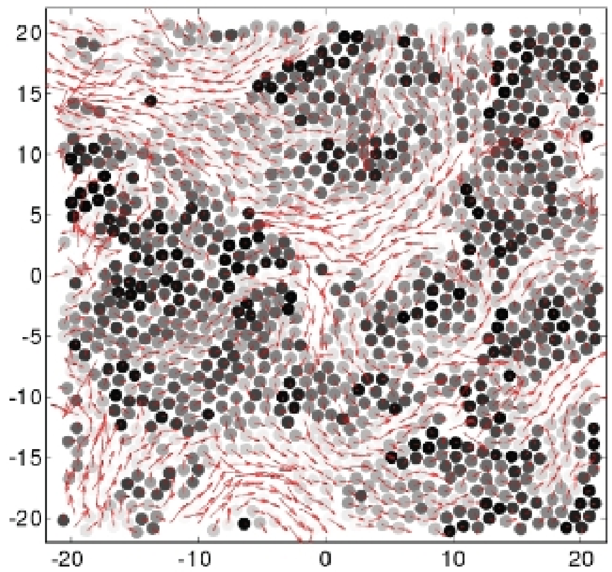

A completely frozen structure

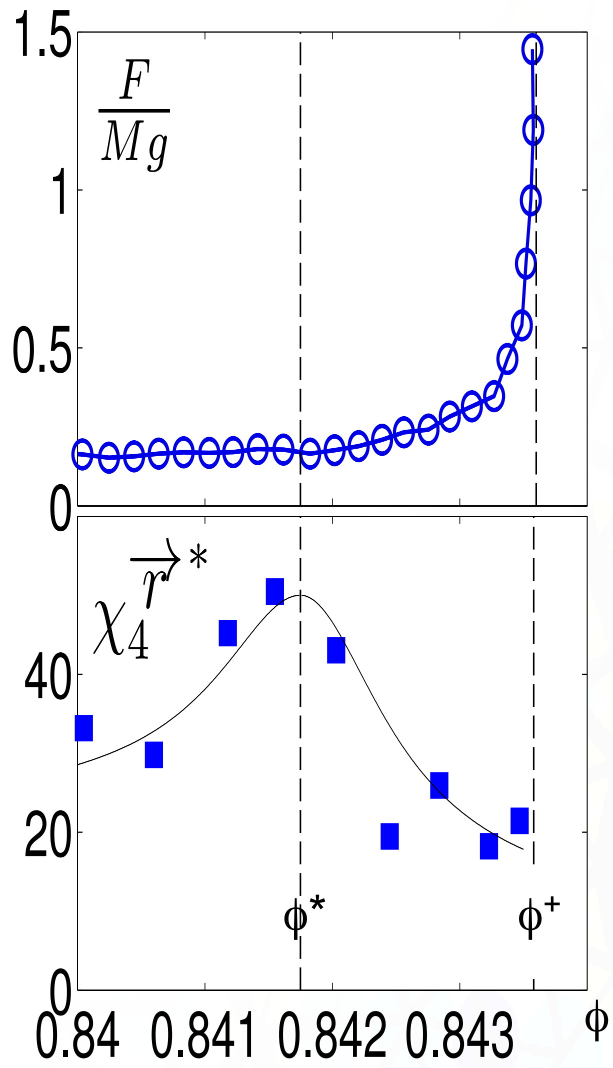
=> *A granular glass*

# Dynamics: Heterogeneous tiny displacements

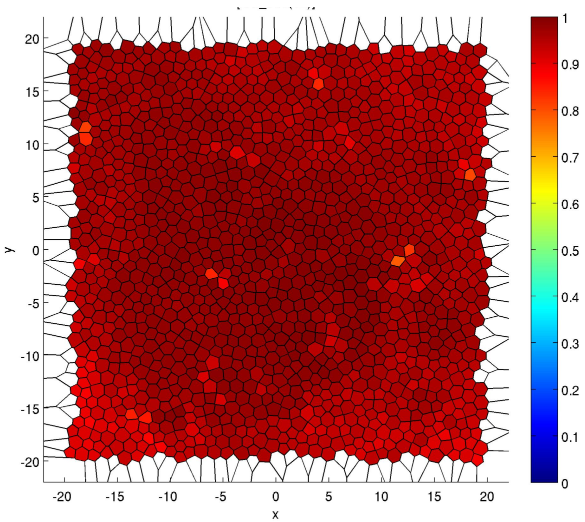
- Particles trajectories : $\vec{r}_i(t)$
- Displacement : $\Delta\vec{r}_i(t,\tau) \equiv \vec{r}_i(t+\tau) - \vec{r}_i(t)$

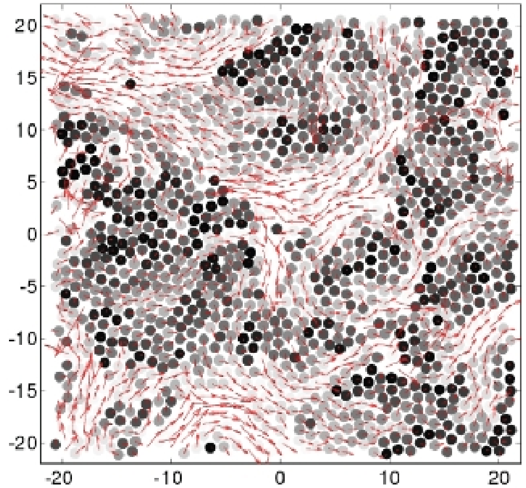


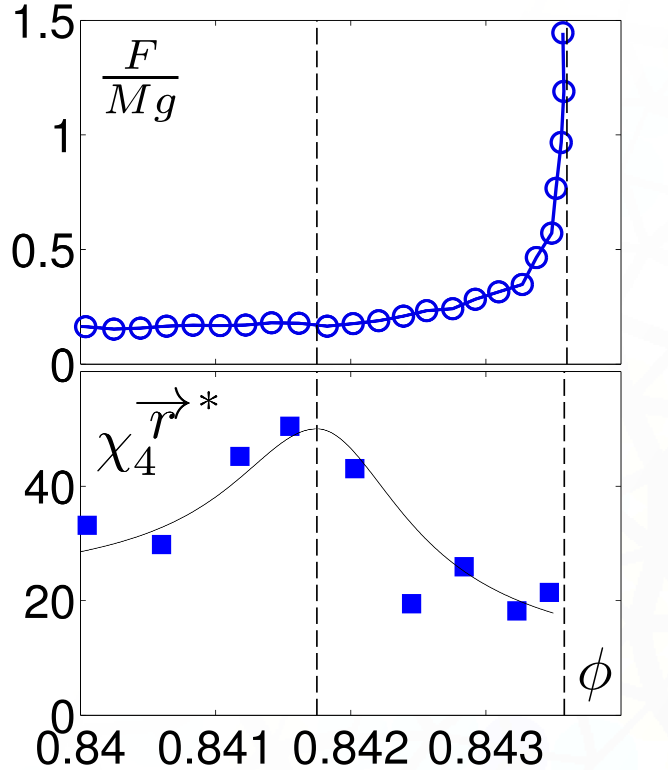




# Altogether…






a frozen structure BUT dynamical heterogeneities

Albeit of a very different kind :
no cage jumps
no change of neighbours
$a^* = 5.10^{-3}$ d

- What is the mechanism responsible for such heterogeneities?
- Why is there a maximum and not just a divergence ?

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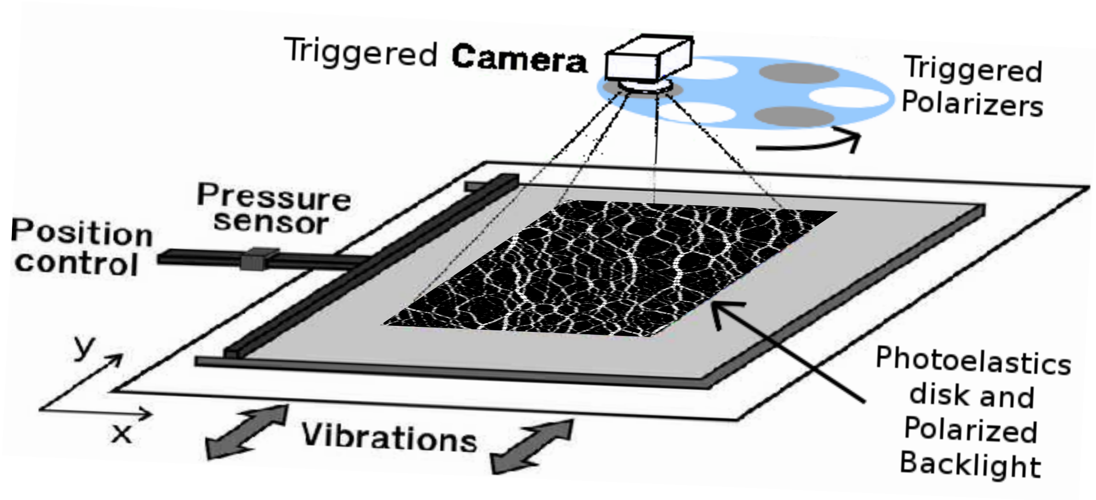
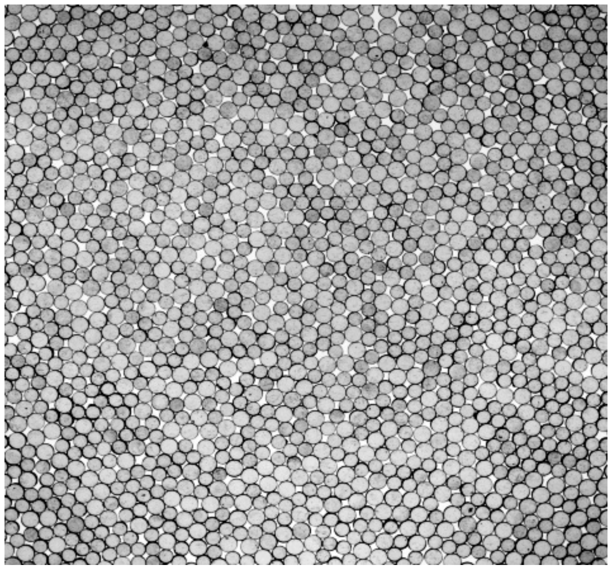
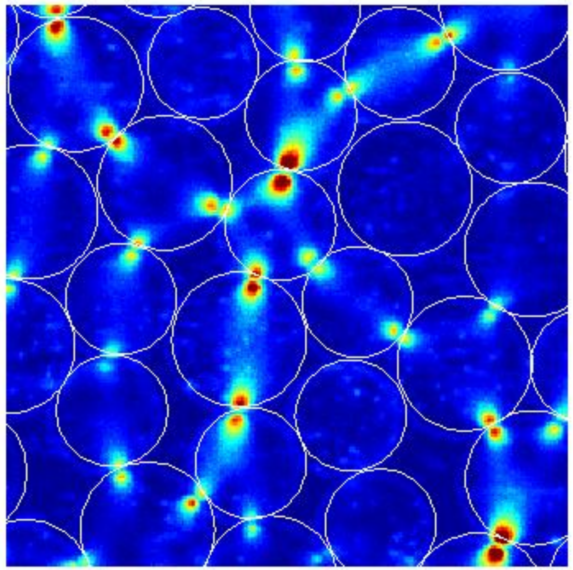
# Redo the experiment with soft photoelastic discs => access to contacts

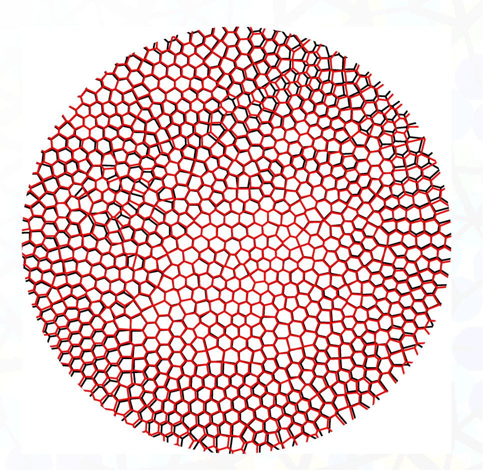


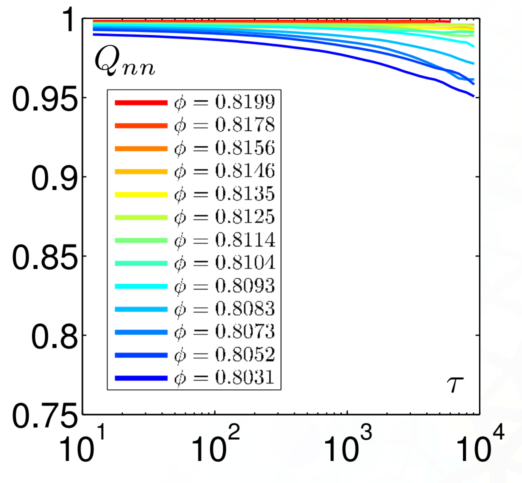





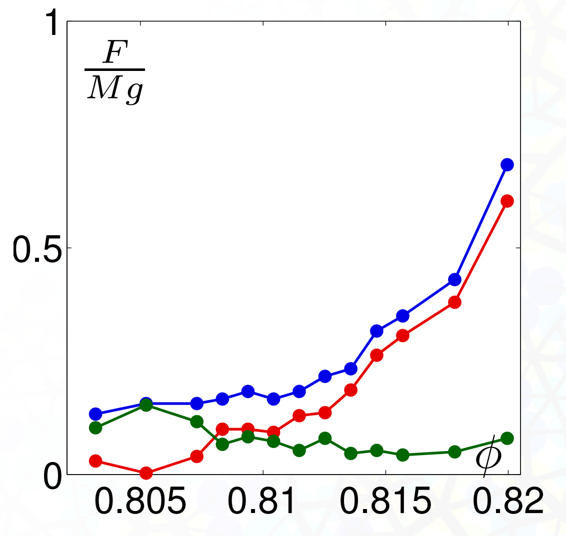
# Same protocole: again a granular glass

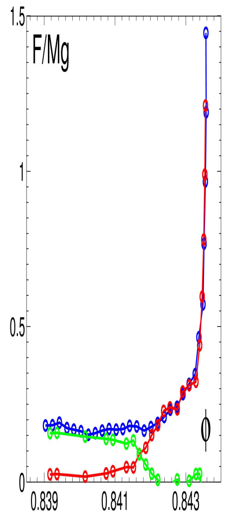
- A frozen structure





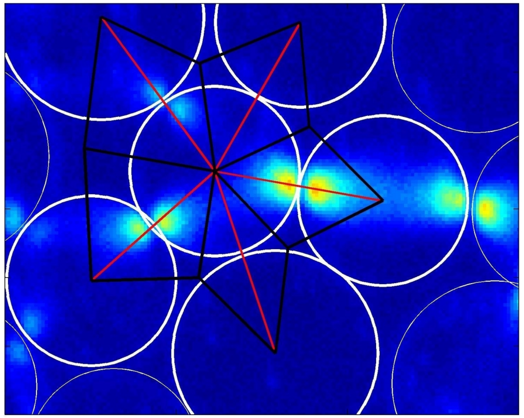

- But this time a glass of soft discs



# Signature of jamming within contacts

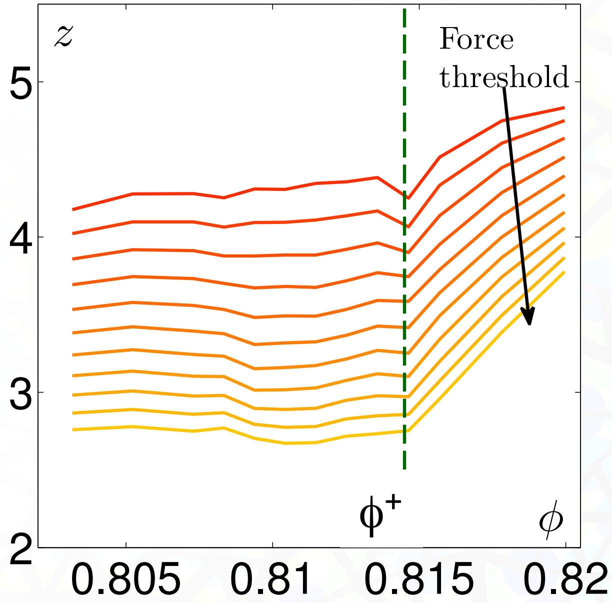
Interparticle force measurement

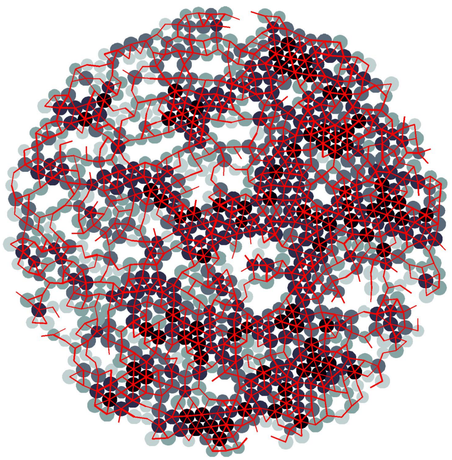


thresholding gap $< \epsilon$

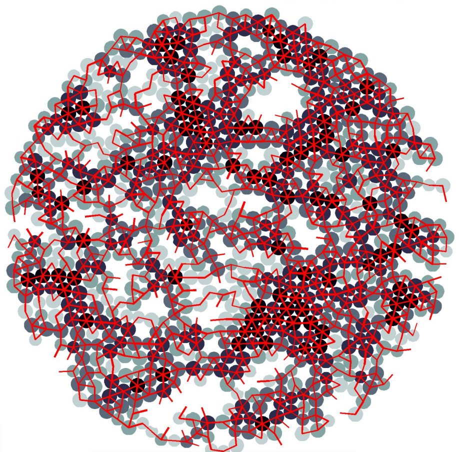
$\rightarrow$

thresholding force $> f_0$

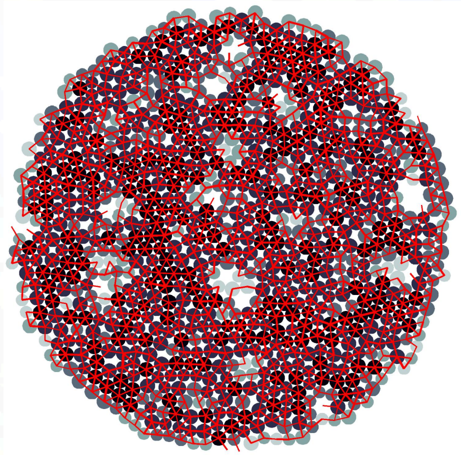


$\phi < \phi^+$



$\phi^+ = 0.814$



$\phi > \phi^+$

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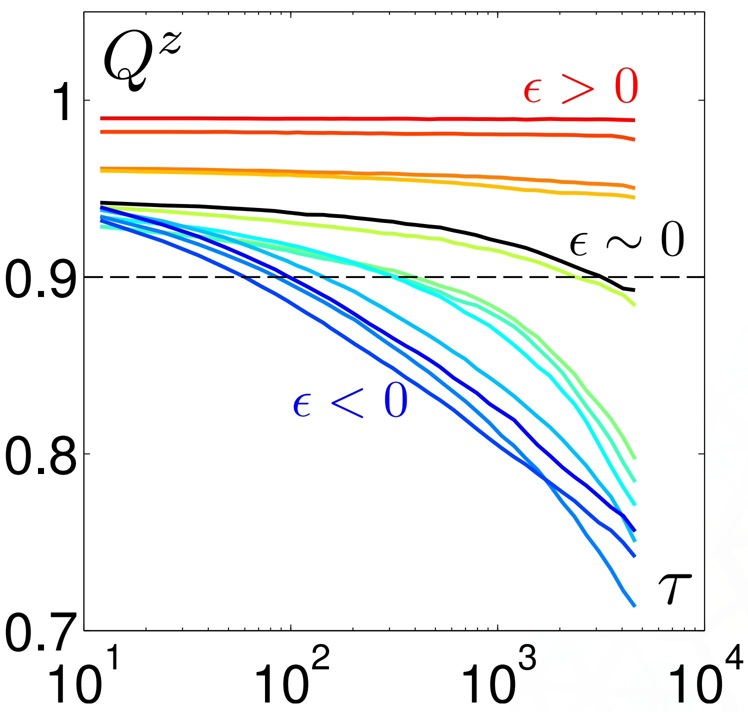
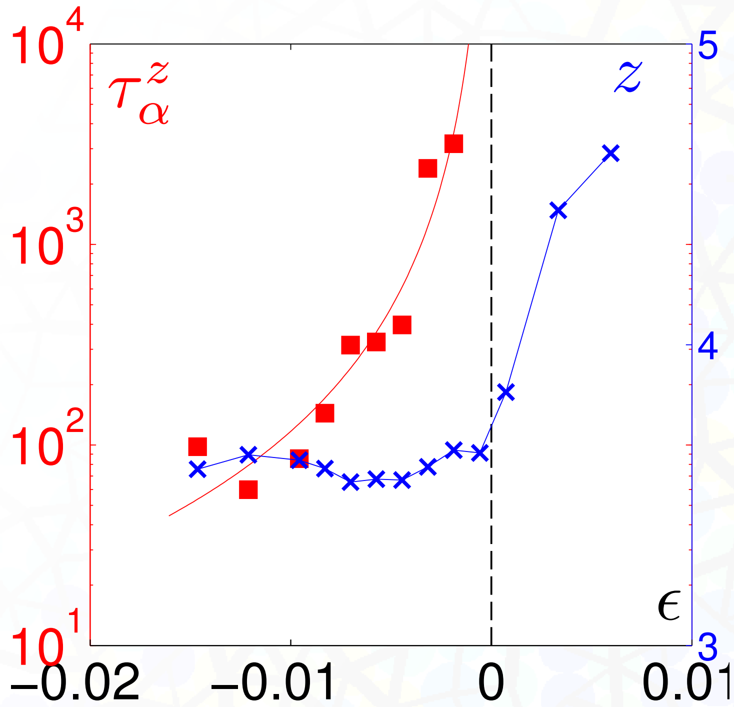
# Dynamics of the contact network…

$$Q^z(t,\tau) = \frac{1}{N}\sum_i Q_i^z(t,\tau) \quad \text{where } Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases}$$

$$Q_z(\tau) = \langle Q^z(t,\tau)\rangle_t$$

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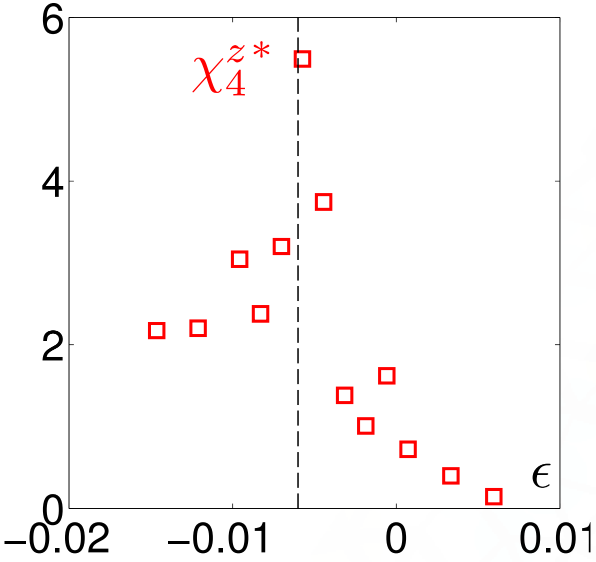
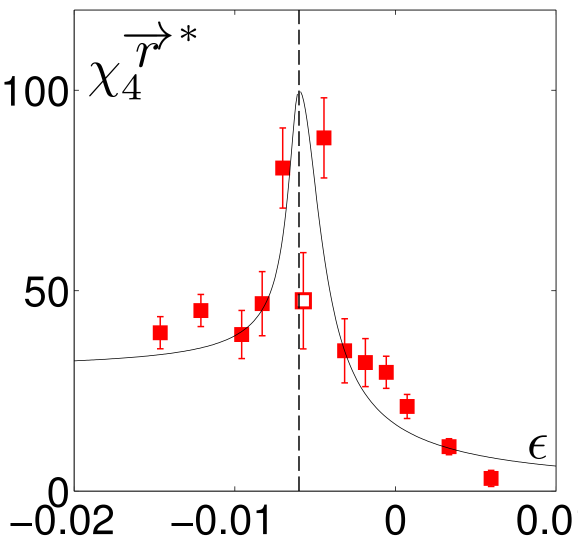
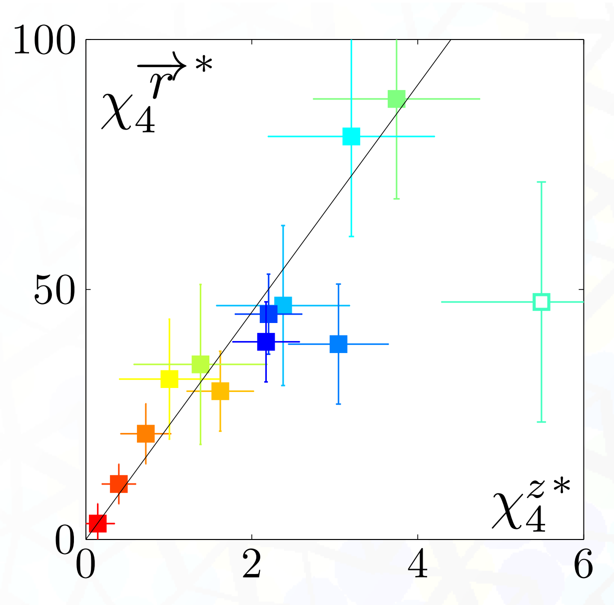
# …is heterogeneous and governs the grains motion

$$\chi_4^{z,r}(\tau) \equiv N Var\left(\left\langle Q_i^{z,r}\right\rangle_i\right)_t \quad \left\{ \begin{array}{l} Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases} \\ Q_i^r(t,\tau) \equiv \exp\left(-\dfrac{\Delta r_i^2}{2\left\langle \Delta r_i^2 \right\rangle}\right) \end{array} \right.$$

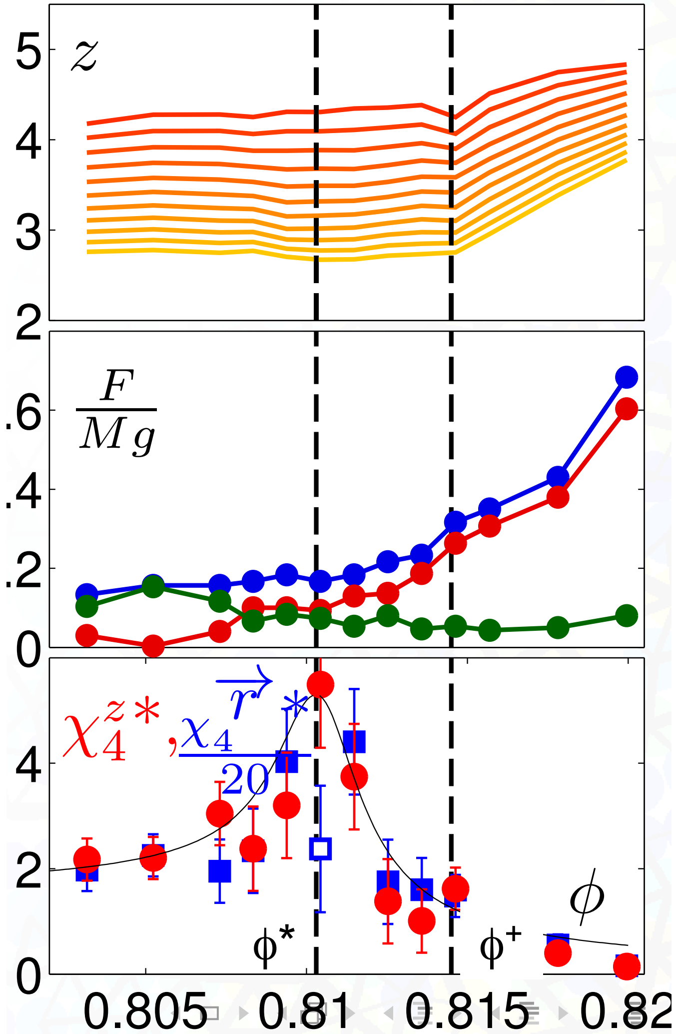


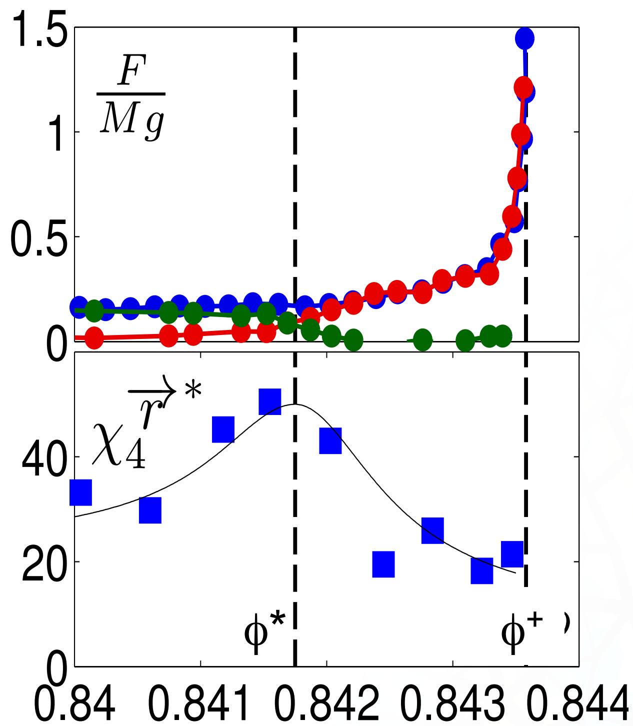



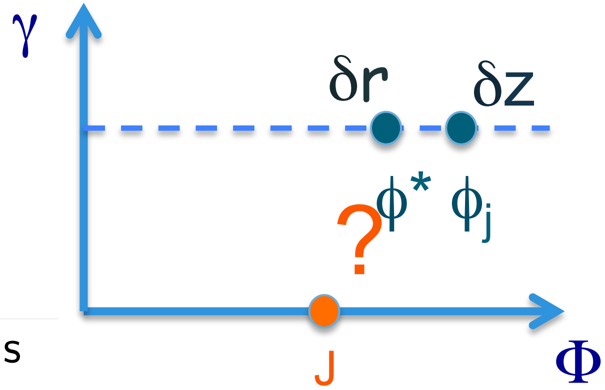
# Summary: two distinct signatures

Soft (Photoelastic) grains

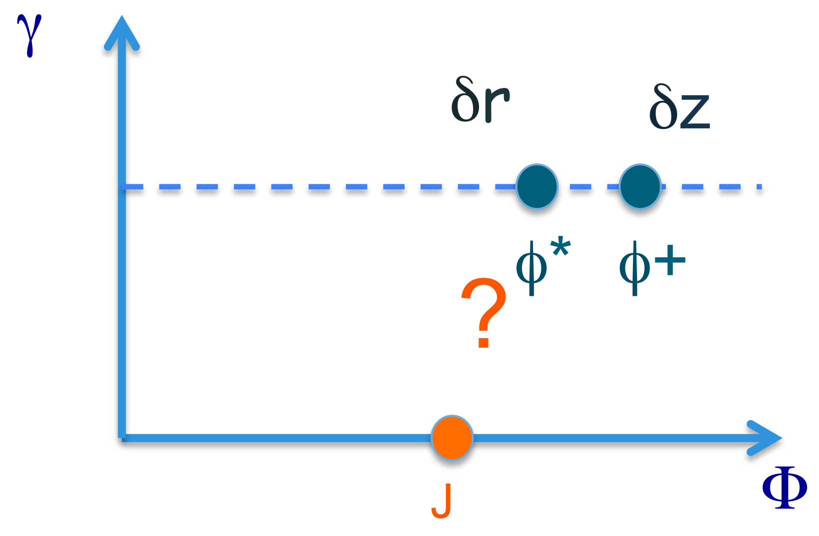



Hard (Brass) Grains



# Reducing the vibration

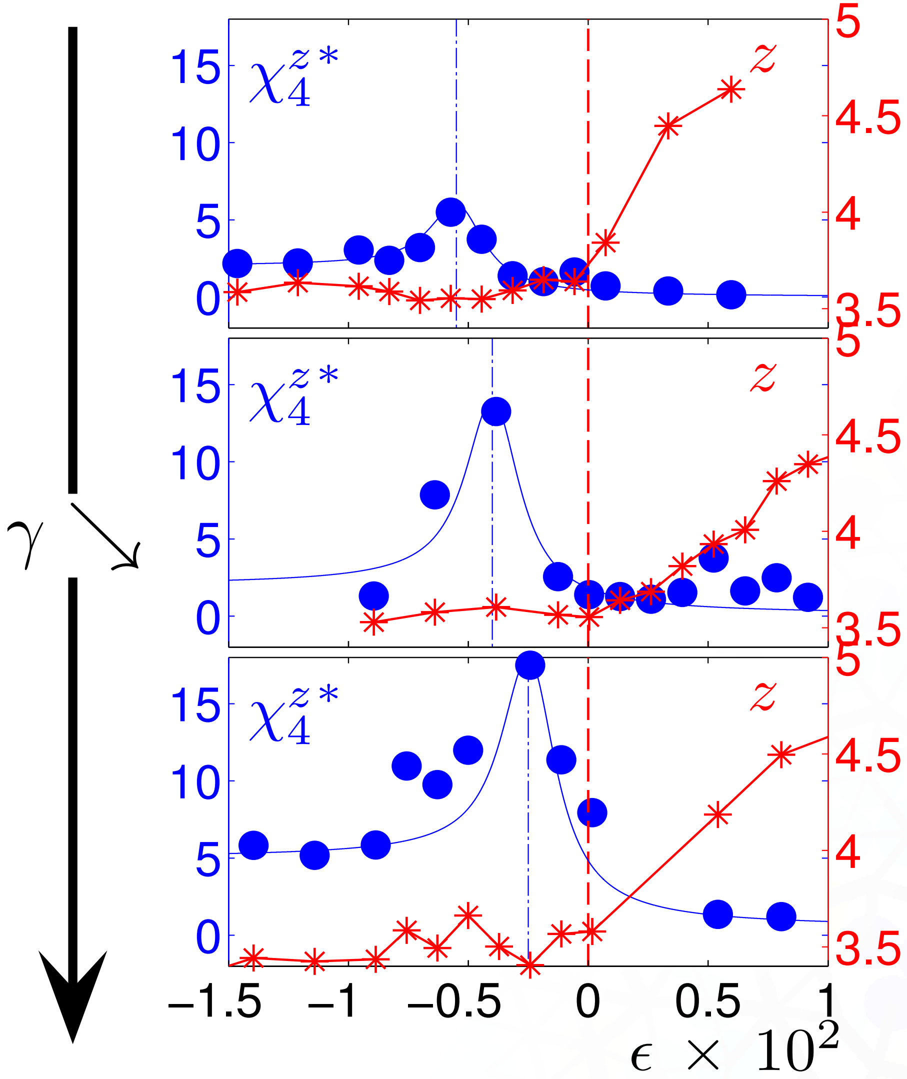



**Three sets of experiments**

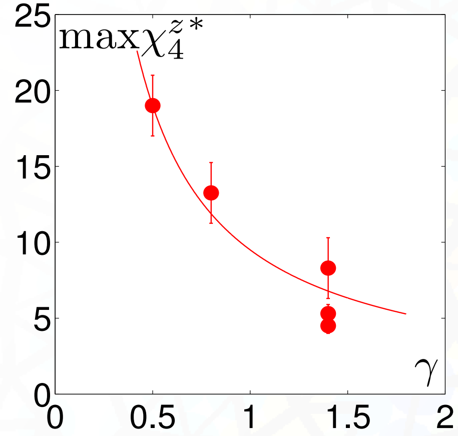
$f = 6.25, 7.50, 10.00\,\mathrm{Hz}$
If $f < f_0 = 4.17$ Hz, no motion.

$$\gamma = \frac{f - f_0}{f_0}$$

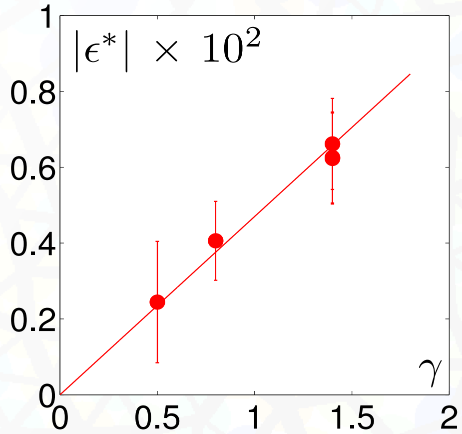
# Decreasing the vibration



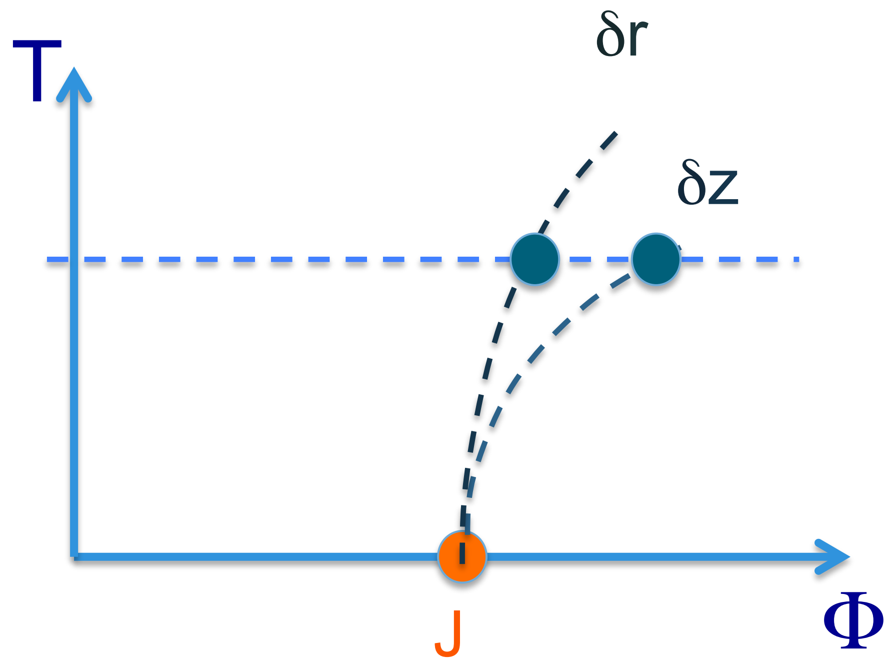

The fluctuations increase

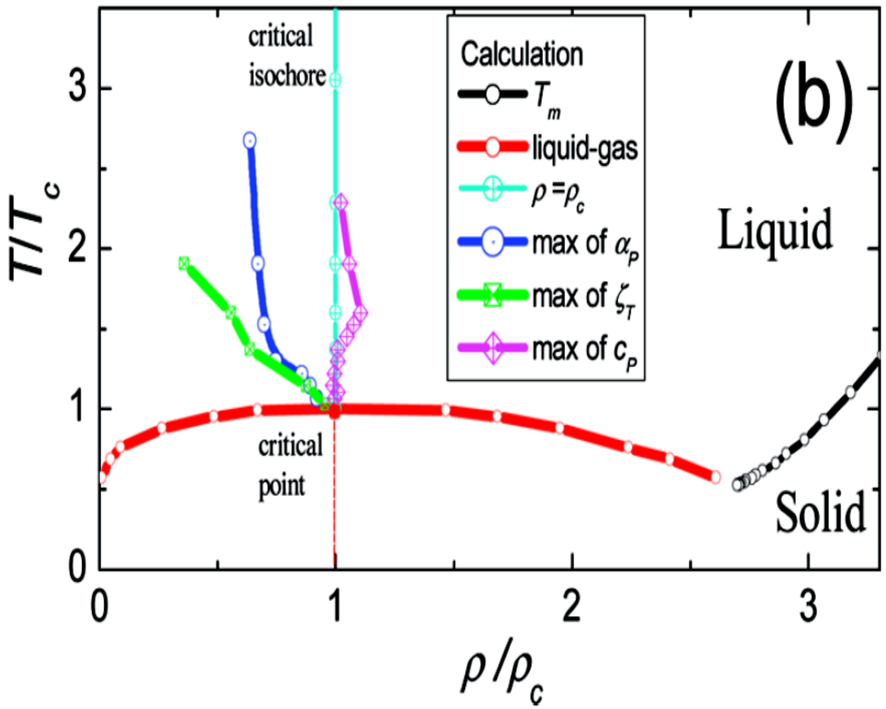



The crossovers merge



# Hence two crossover lines : Widom lines



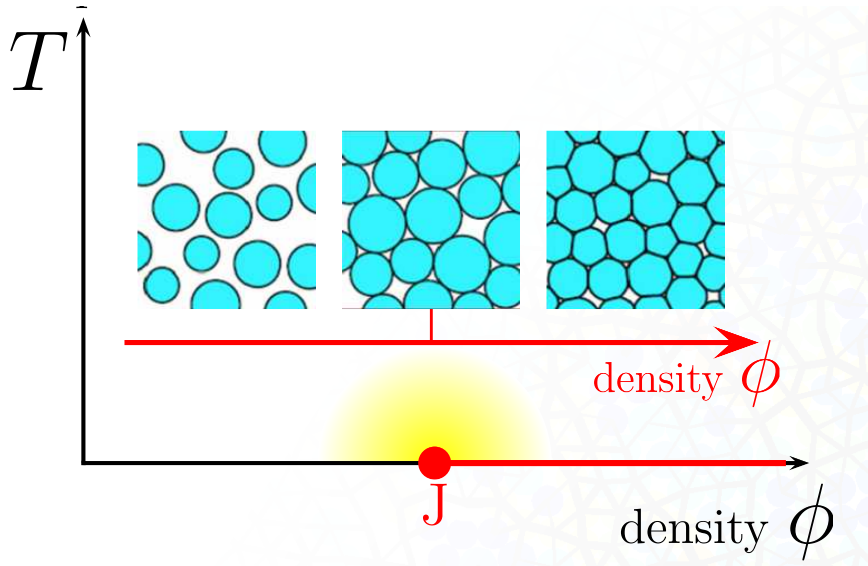



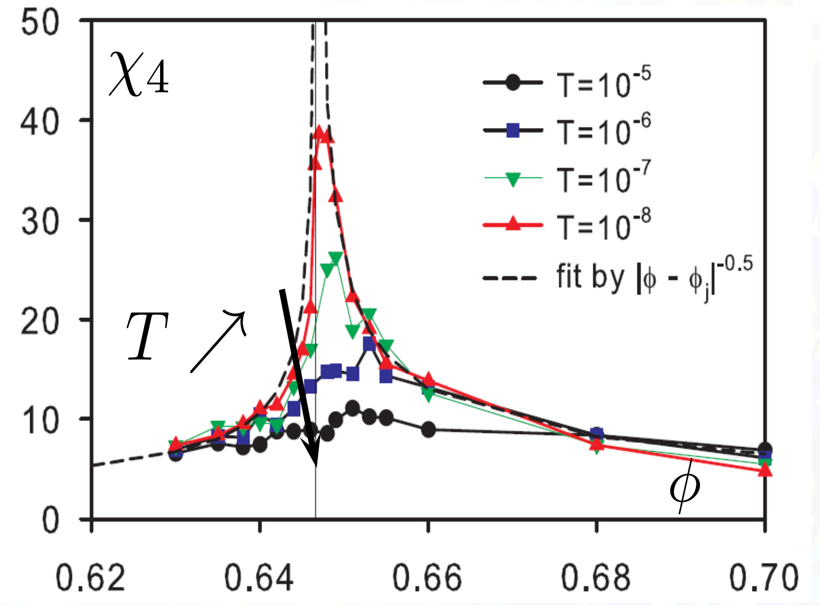

**How far from the critical point ?**

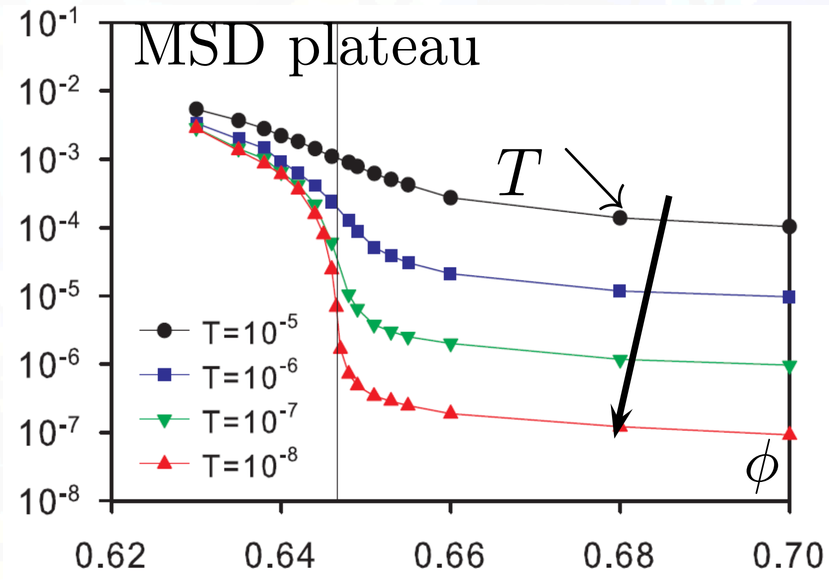
# Comparison with thermal soft spheres…

Simulation of thermal soft-spheres

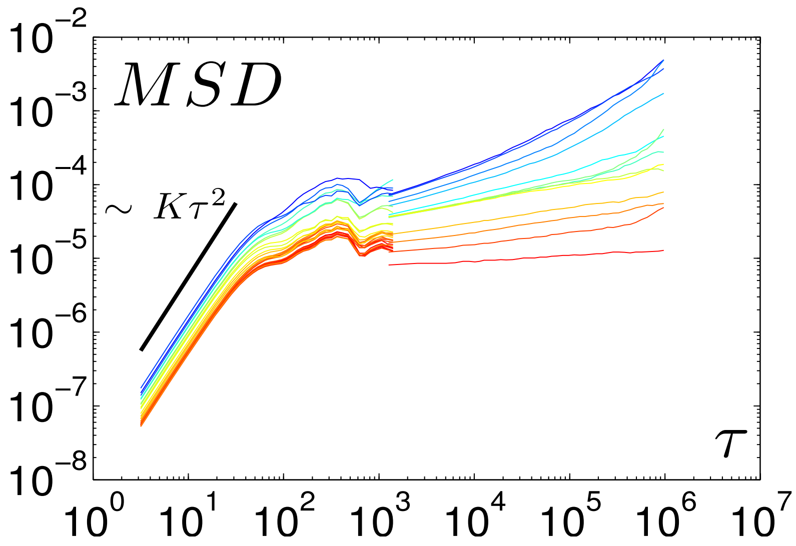
Ikeda et al, 2012

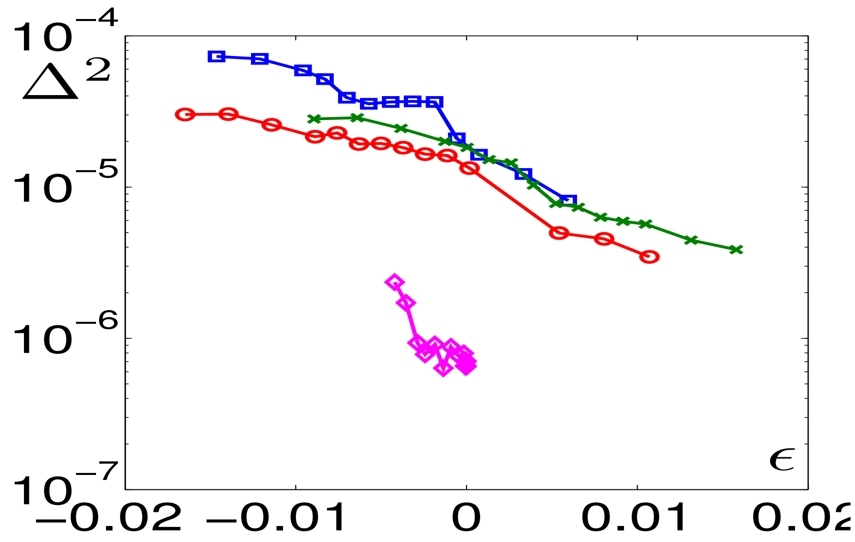


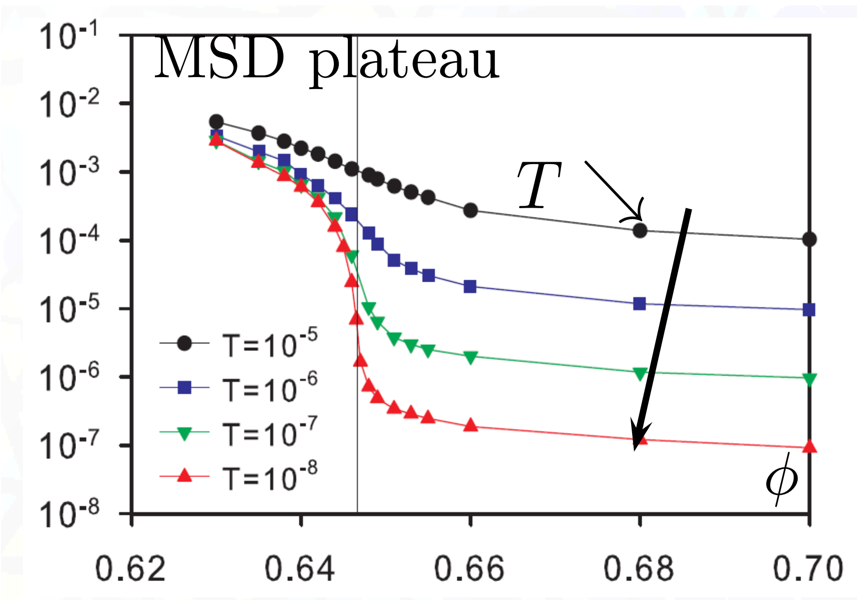



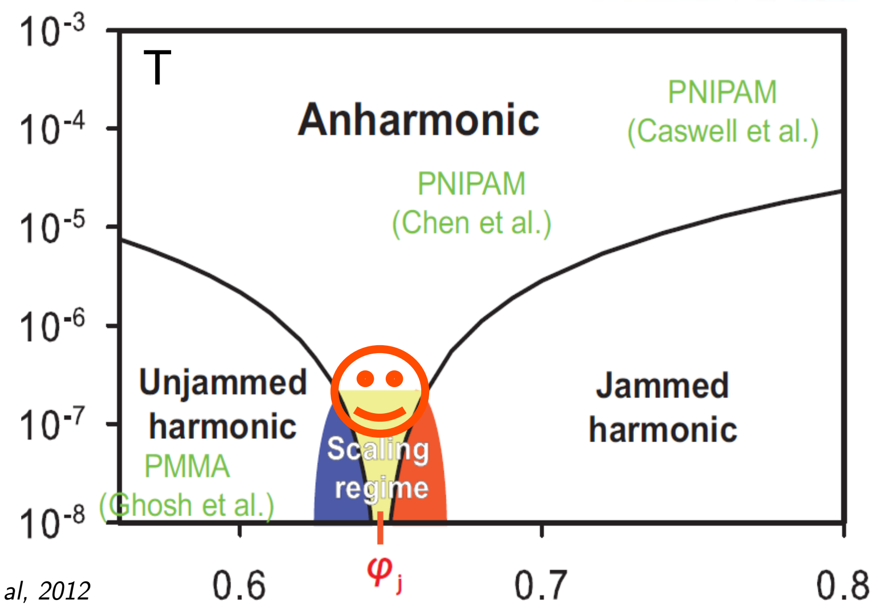



# Discussion: in the light of the street-lamp



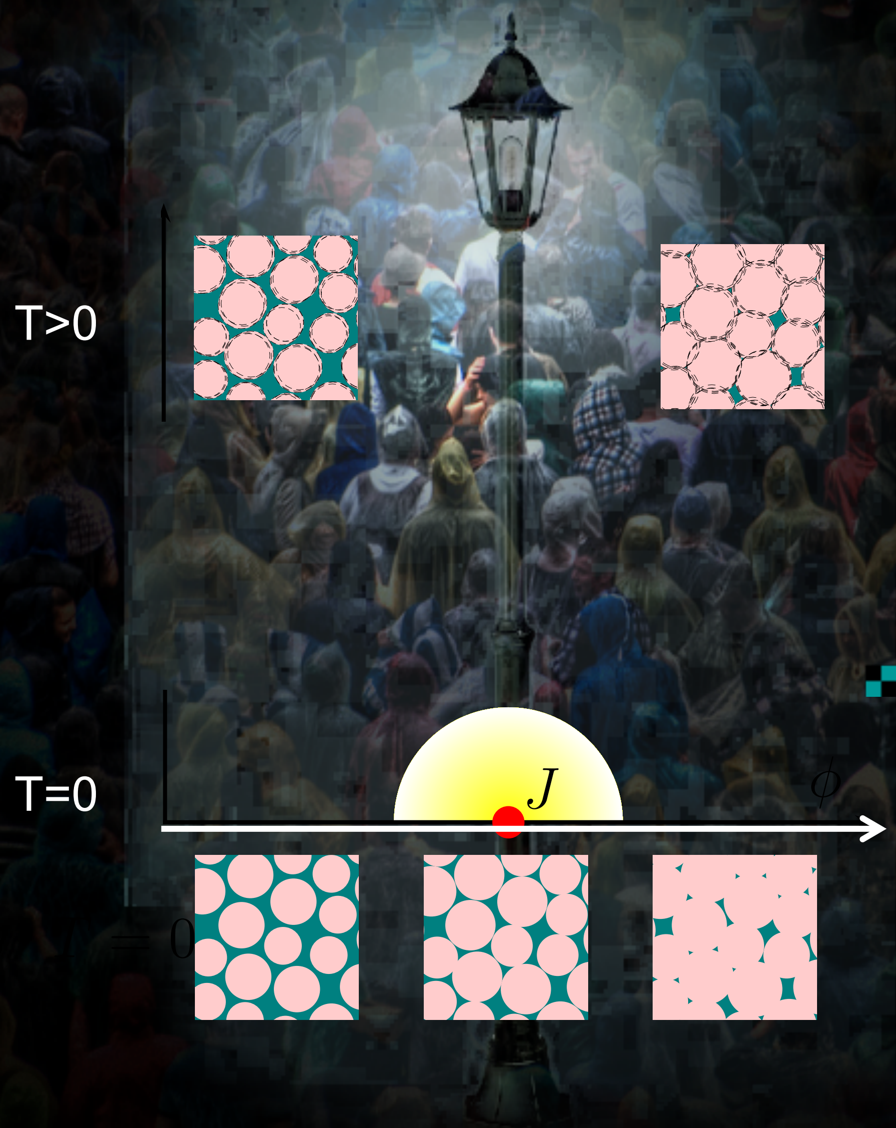




al, 2012

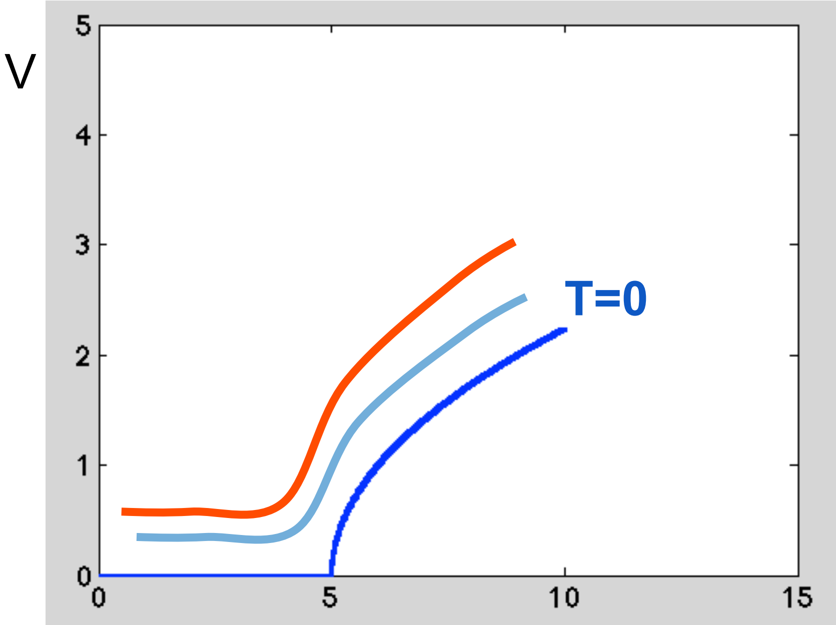
# Conclusion of the first part

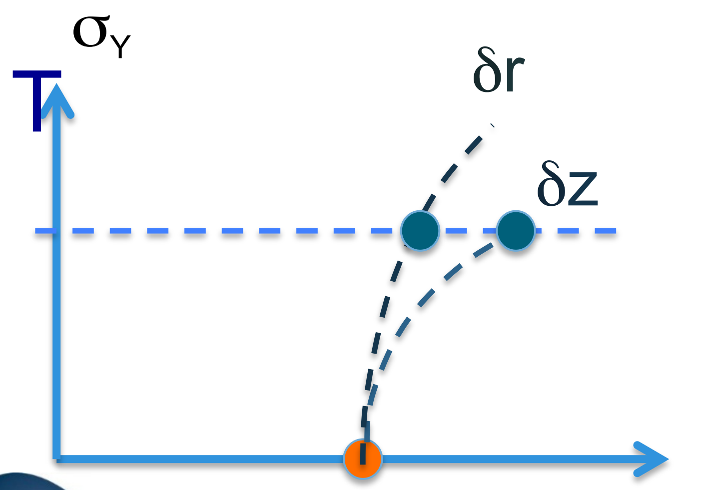


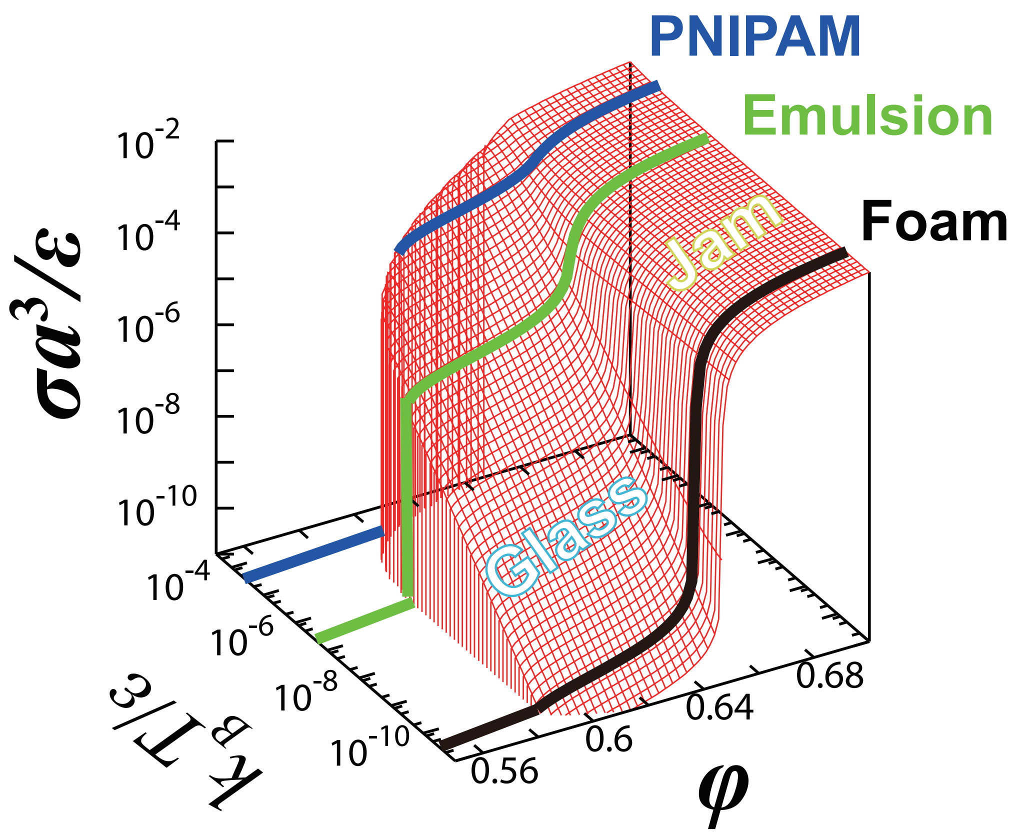

- Shaken Granular Experiments are in the street lamp halo of the J point:
  - They can constrain existing theories
  - Theories have something to say about the real world…
  - One cannot exclude effects of friction at the quantitative level

- => One step further (in the dark…)
  - Yielding close to jamming
  - A first attempt to probe elasticity close to jamming

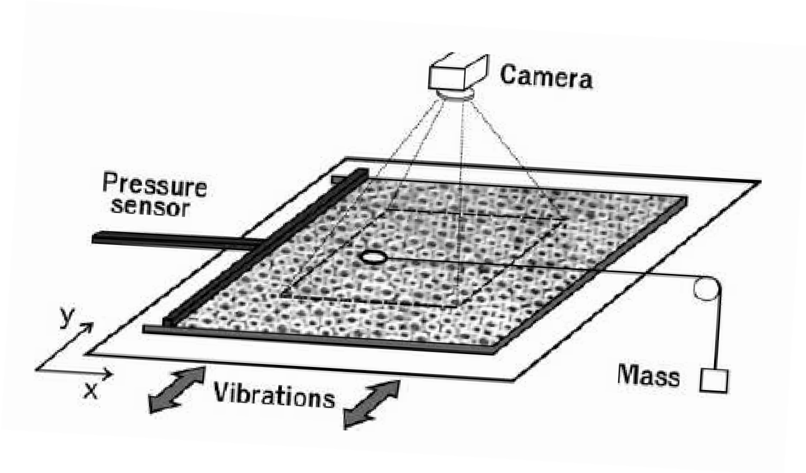
# Yielding close to jamming …

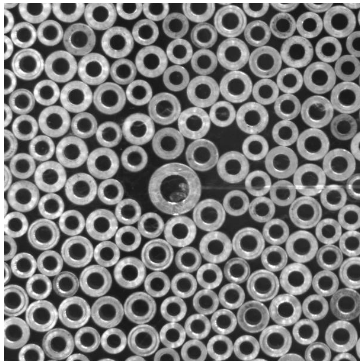


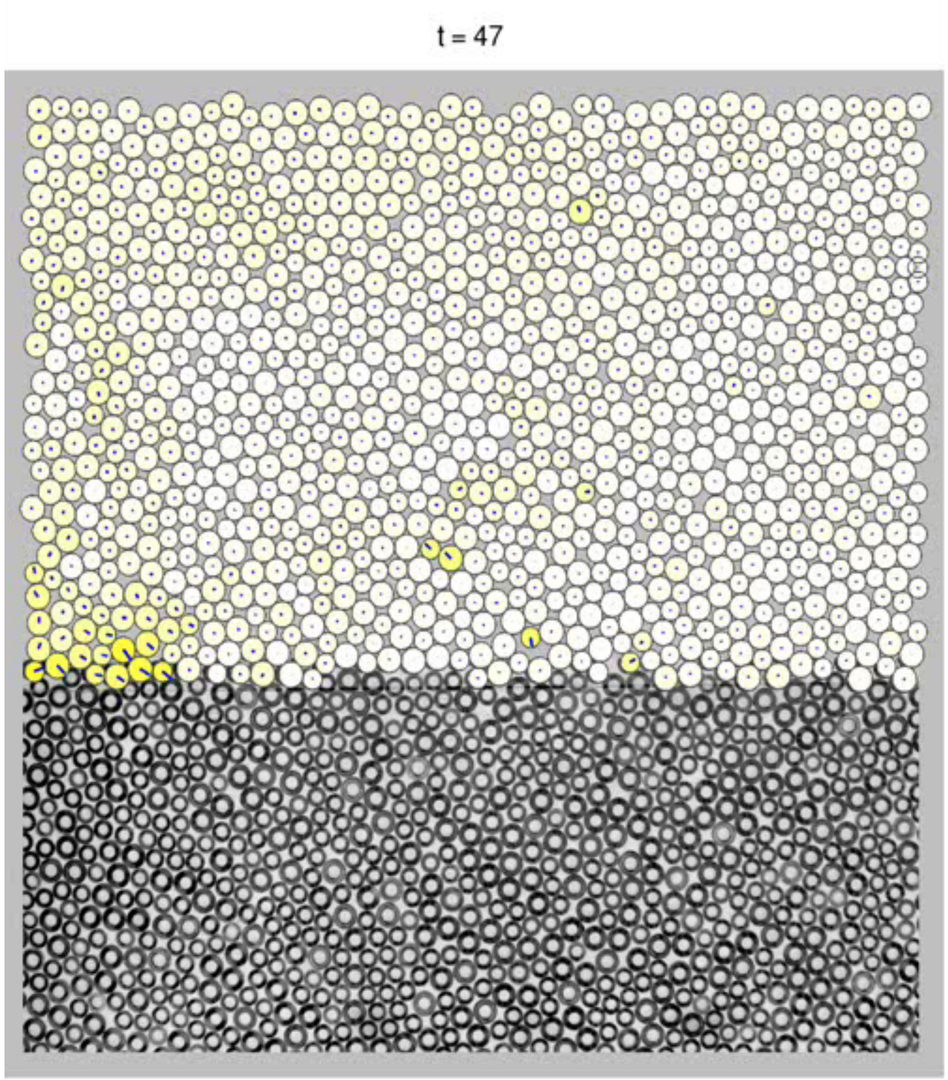



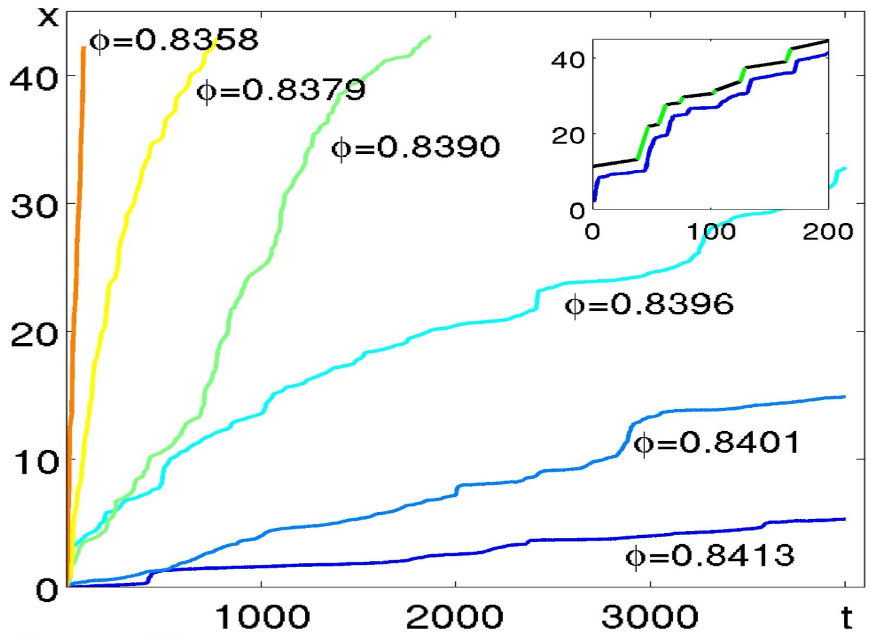
# Yielding close to jamming : the motion of an intruder …

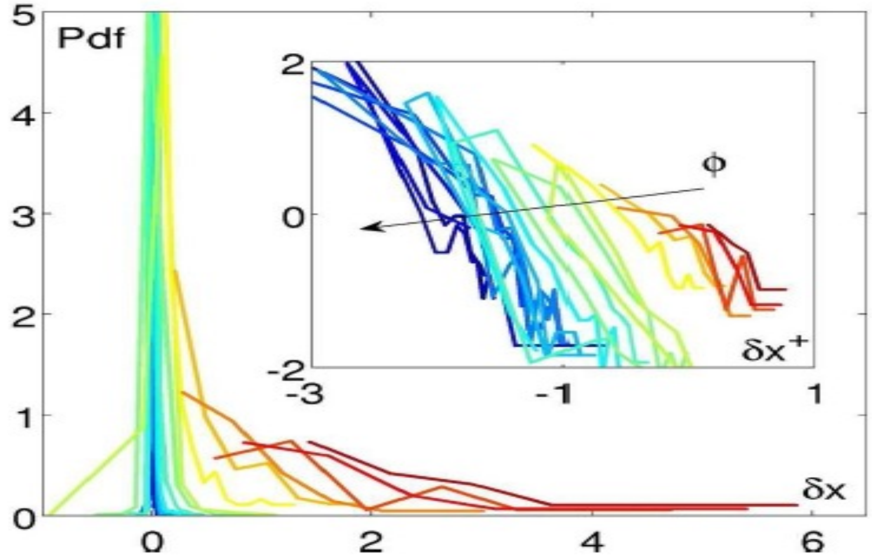


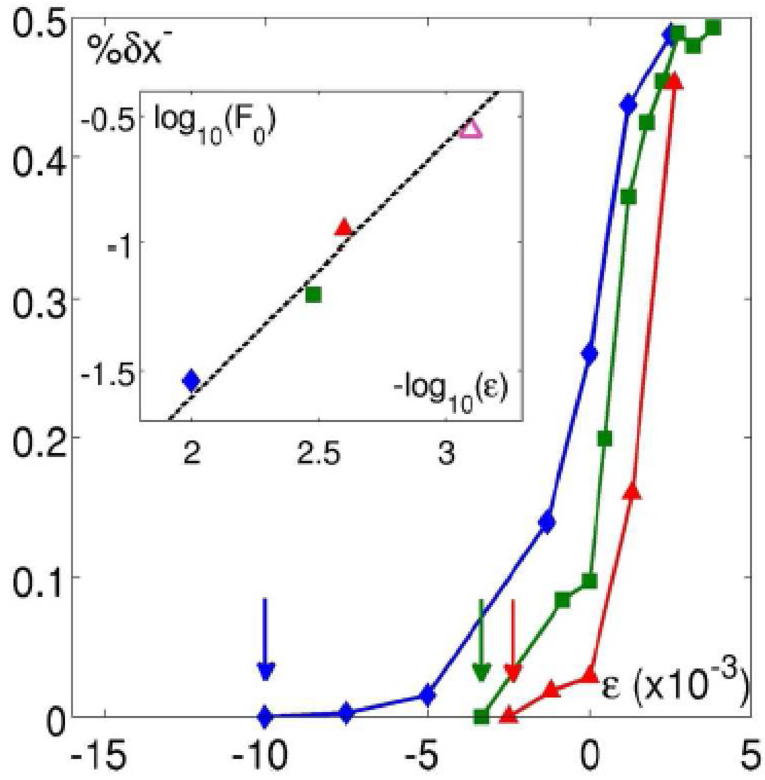


# Evidence of a fluidization transition

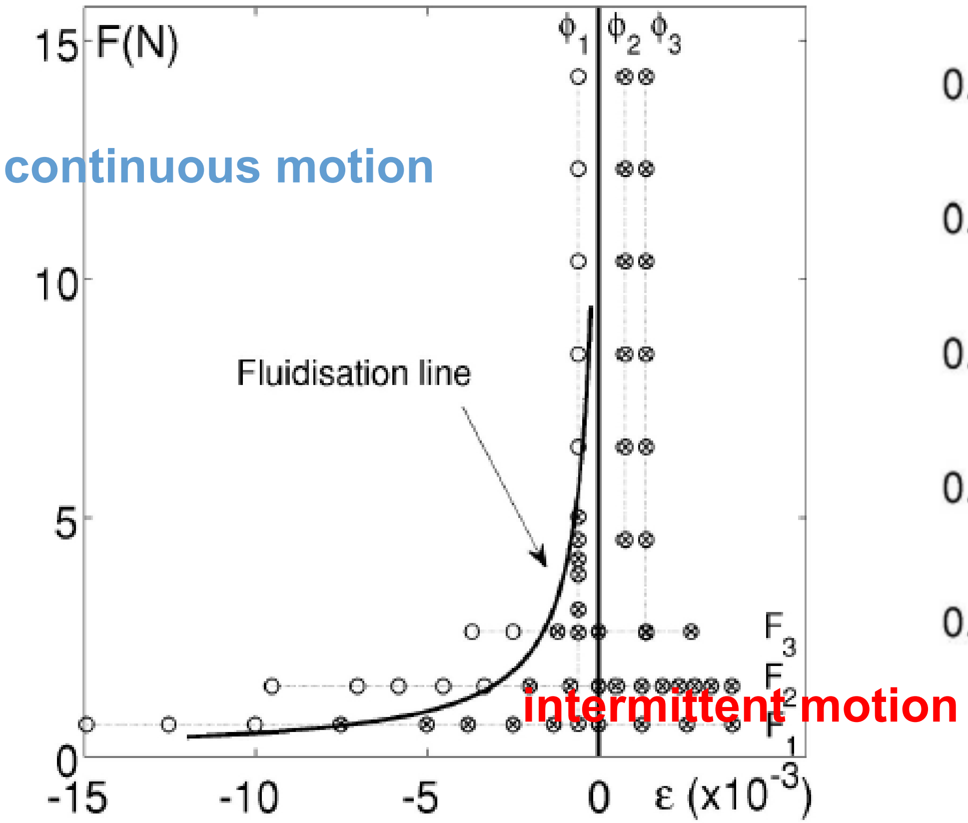


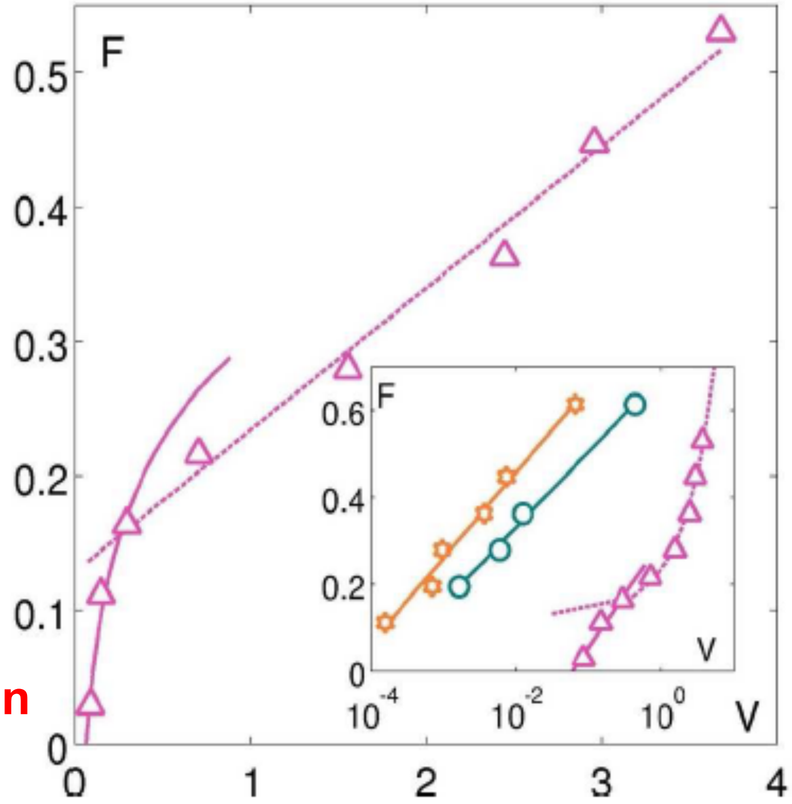




Transition : % δx < 0 --> 0



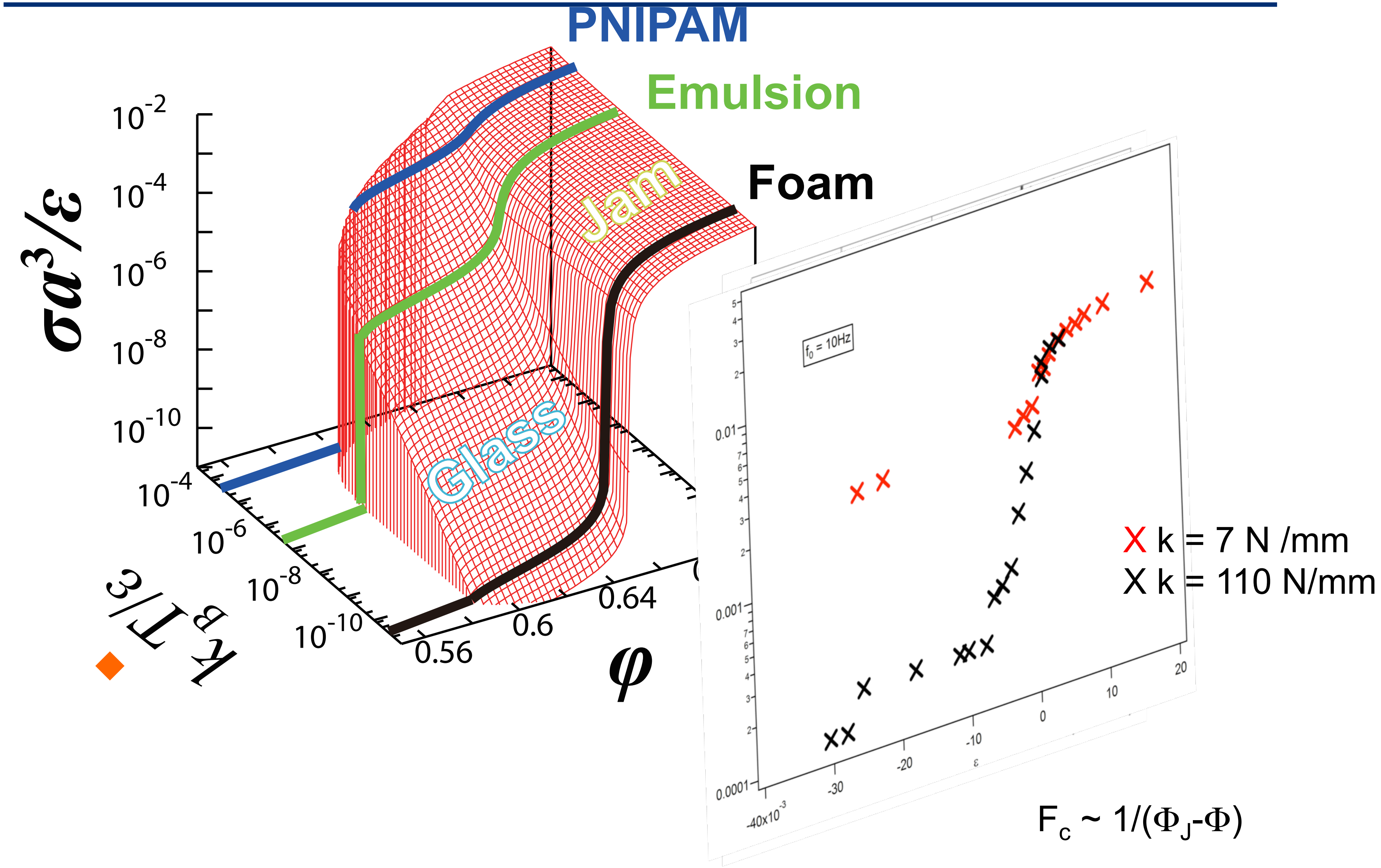
# Indeed two very different rheological behaviors




- Fluidized regime : F ∝ <V> :
- Intermittent regime : F ∝ ln <V>

# Critical force : “thermal” yield stress




$F_c \sim 1/(\Phi_J - \Phi)$

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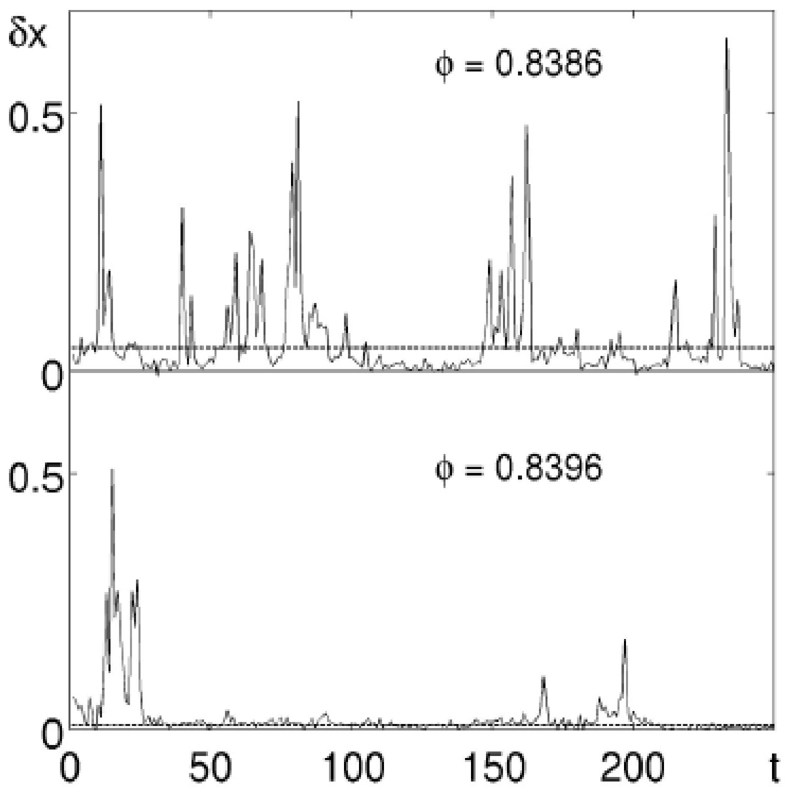
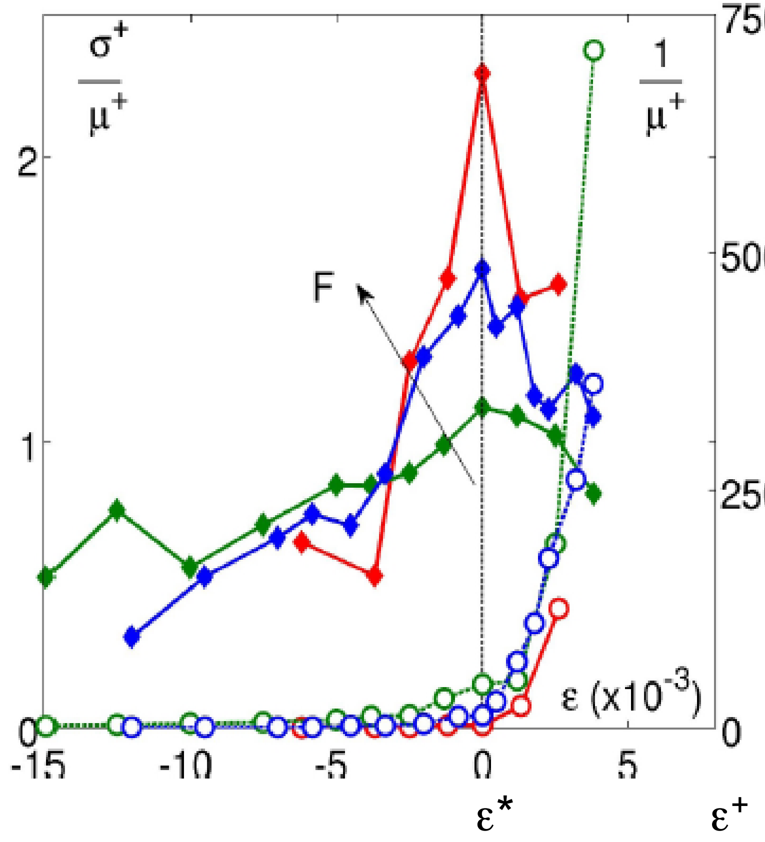
# In the intermittent regime : signature of Jamming



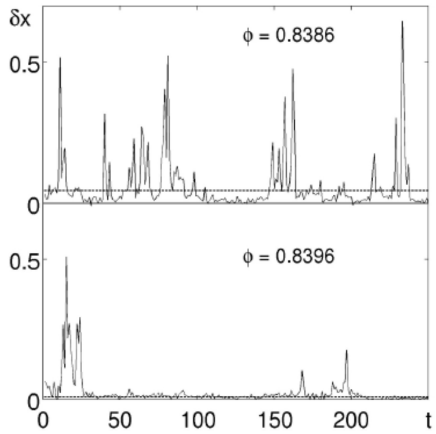



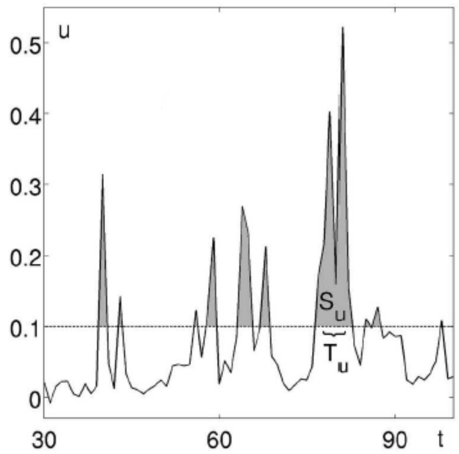

$$\mu^+ = \langle \delta x^+ \rangle$$

$$\sigma^+ = \left( \langle \delta x^{+2} \rangle - \langle \delta x^+ \rangle^2 \right)^{1/2}$$

**Independant signature of $\Phi^*$**

# Pinning-depinning like dynamics => Crackling noise signals

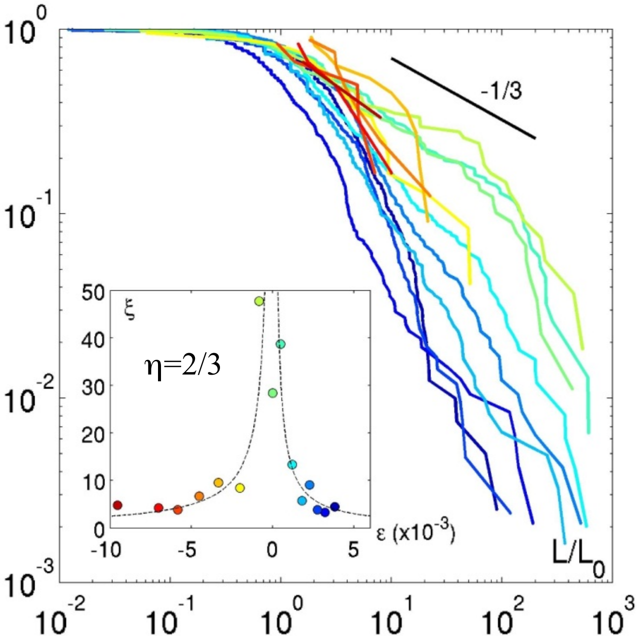


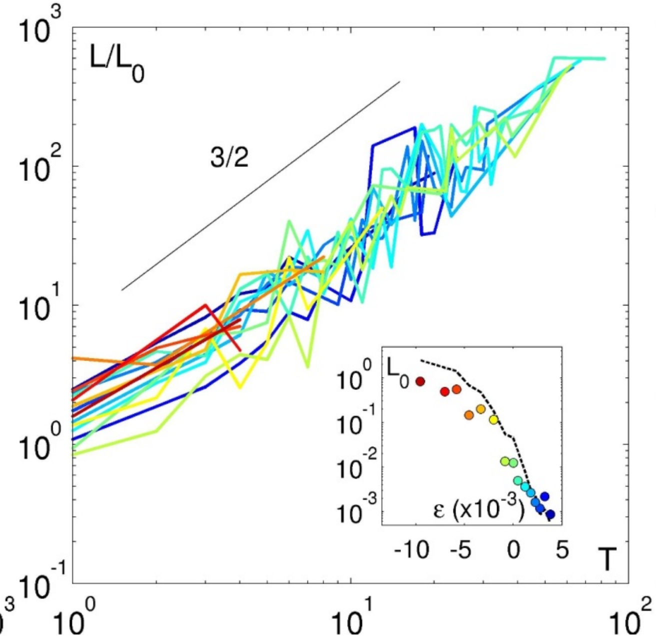



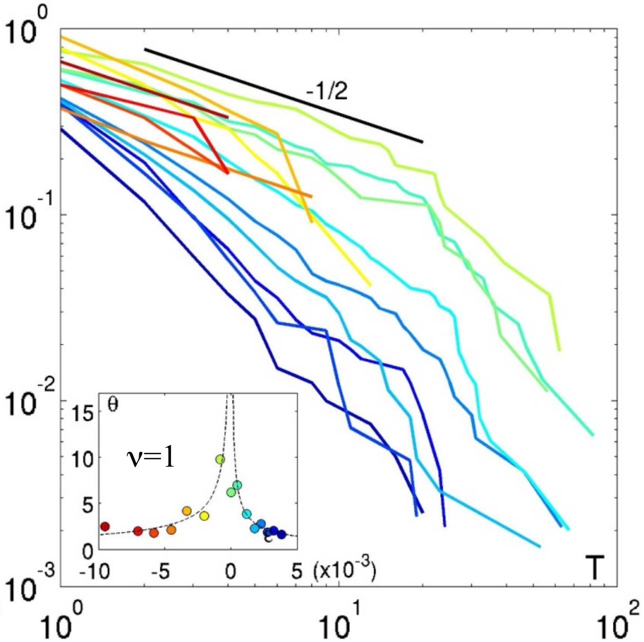

$$pdf(T) \propto T^{-(1+\alpha)} f\left(\frac{T}{\theta(\phi)}\right);\ \theta(\phi) \propto \left|\phi-\phi_J\right|^{-\eta}$$

$$L \propto T^{1/z}$$

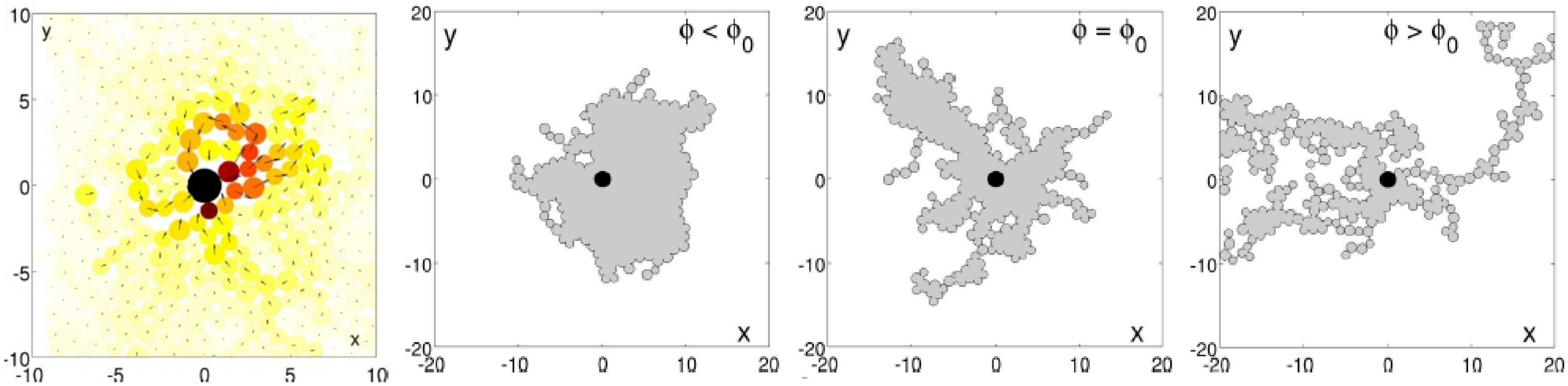
$$pdf(L) \propto L^{-(1+\beta)} f\left(\frac{L}{\xi(\phi)}\right);\ \xi(\phi) \propto \left|\phi-\phi_J\right|^{-\nu}$$



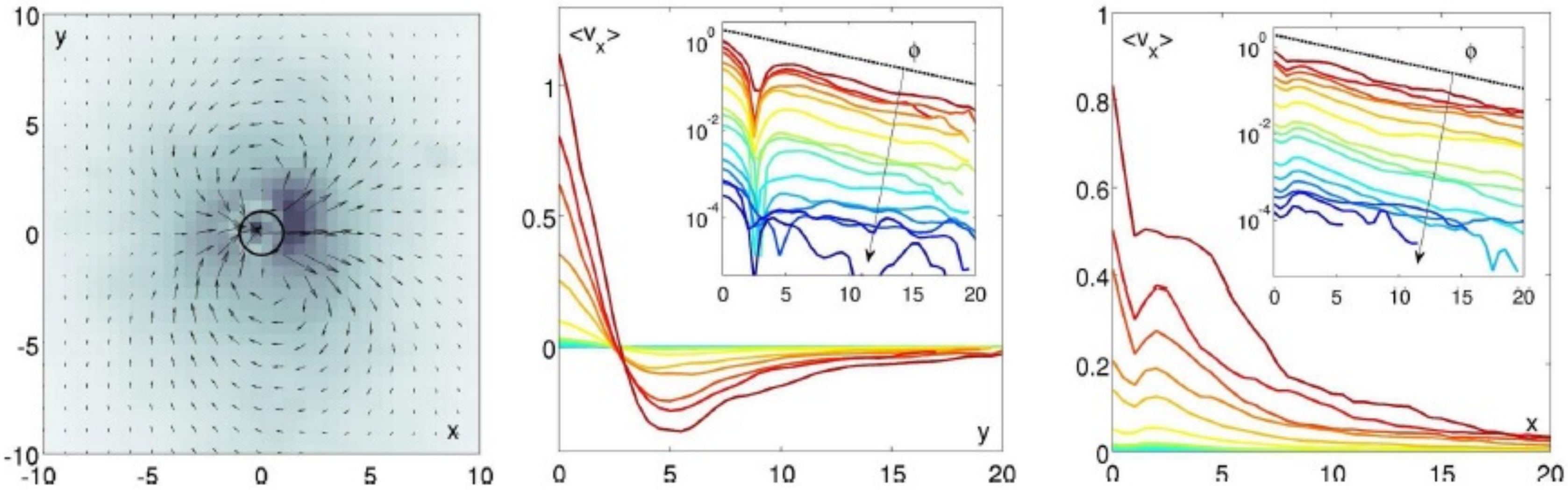
# Instantaneous field around the intruder




Typical contours of clusters of the 15% fastest particles

- A very heterogeneous and intermittent field
- More and more chained-like clusters
  - $\langle n_{neigh} \rangle$ goes from 4 to 5.5
  - fractal dim. of the contour goes from 1.3 to 1.5

# Averaged displacement field around the  intruder




- Two symmetric vortices on  both side of the intruder

- No sharp evidence of the transition in the averaged field

- Exponential decrease of $\langle v_x \rangle$ with distance from the intruder :

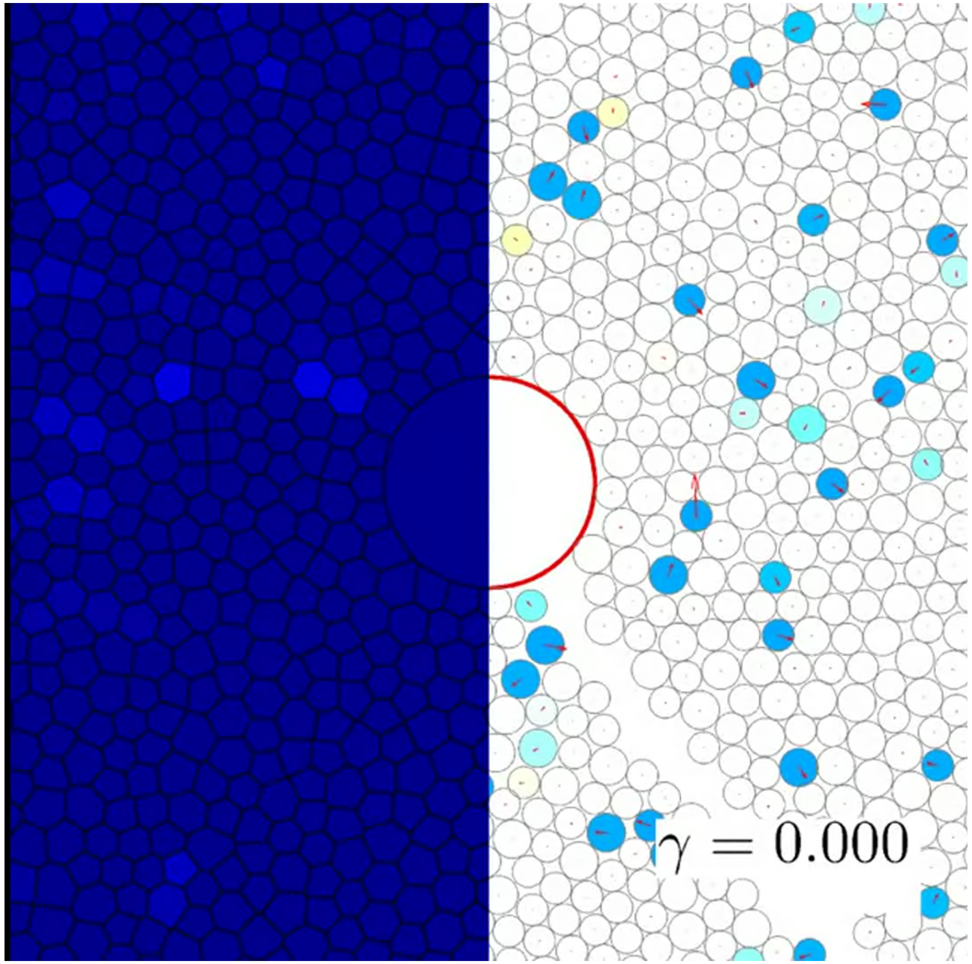
the associated length scale does not depend on $\Phi$.

# Probing elasticity : set up

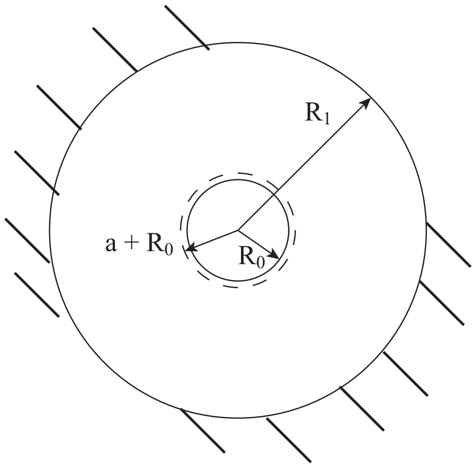
- Prepare the system at large packing fraction under vibration
- Inflate an intruder in the center (the vibration is stopped)
- Decrease the packing fraction while vibrating
- iterate

R0 - > R0 + a

$\gamma$ = a/R0



# Probing elasticity : the linear elastic framework




$$div\left(\underline{\underline{\sigma}}\right)=0$$

$$\underline{\underline{\sigma}}=\frac{1}{2}Tr(\underline{\underline{\sigma}})\underline{\underline{1}}+\underline{\underline{\tau}}$$

$$\underline{\underline{\sigma}}=KTr(\underline{\underline{\varepsilon}})\underline{\underline{1}}+2G\underline{\underline{\gamma}}$$

$$\underline{\underline{\varepsilon}}=\frac{1}{2}\left[\underline{\underline{\nabla U}}+{}^{t}\underline{\underline{\nabla U}}\right]=\frac{1}{2}Tr(\underline{\underline{\varepsilon}})\underline{\underline{1}}+\underline{\underline{\gamma}}$$

$$U(R_0)=a$$
$$U(R_1)=0$$

$$\delta\equiv Tr\left(\underline{\underline{\varepsilon}}\right)=-2\frac{a}{R_0}A$$

$$\gamma\equiv J_2\left(\underline{\underline{\gamma}}\right)\equiv\sqrt{\frac{1}{2}\underline{\underline{\gamma}}\circ\underline{\underline{\gamma}}}=\frac{a}{R_0}B\left(\frac{R_0}{r}\right)^2$$

$$P\equiv Tr\left(\underline{\underline{\sigma}}\right)=KTr\left(\underline{\underline{\varepsilon}}\right)$$

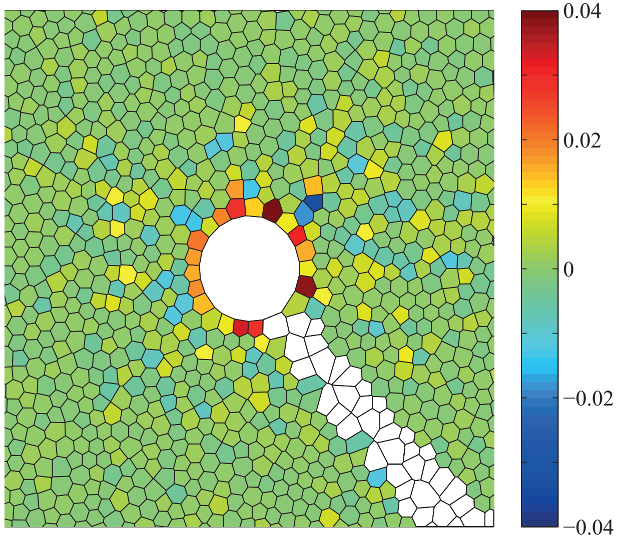
$$\tau\equiv J_2\left(\underline{\underline{\tau}}\right)=\sqrt{\frac{1}{2}\underline{\underline{\tau}}\circ\underline{\underline{\tau}}}=2GJ_2\left(\underline{\underline{\gamma}}\right)$$

$$A=R_0^2\Big/\left(R_1^2-R_0^2\right);B=R_1^2\Big/\left(R_1^2-R_0^2\right)$$

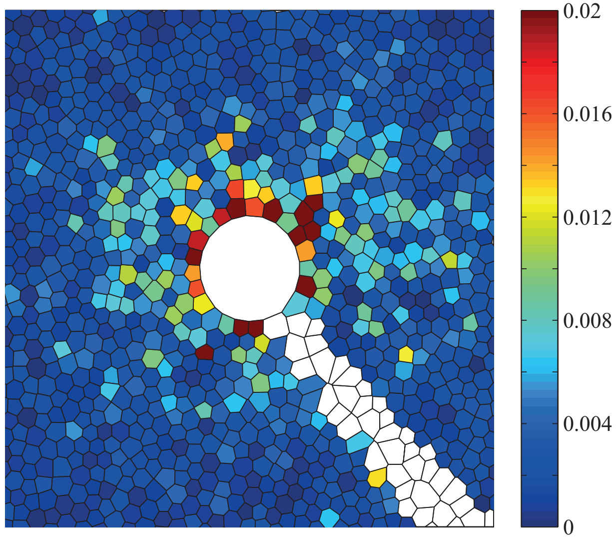
- Nota Bene
  - In the limit of large $R_1$, A->0, B->1 : this is a shear test!
  - G and K are simply obtained by the ratio of the stress and strain tensor invariants

# For each packing fraction and each a/$R_0$

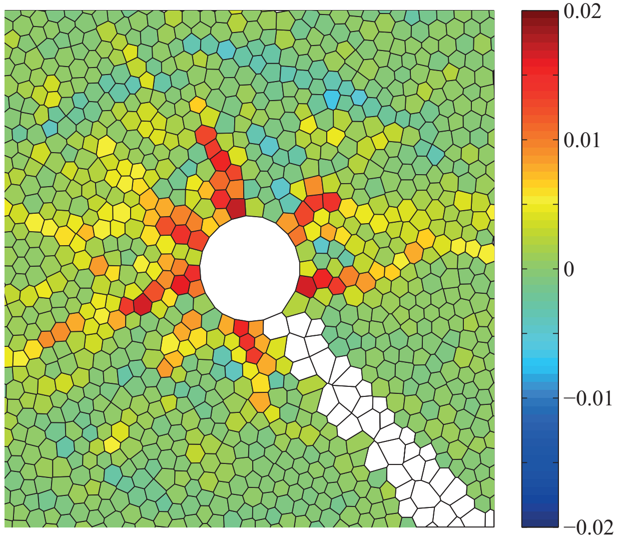
$\delta = Tr(\varepsilon)$



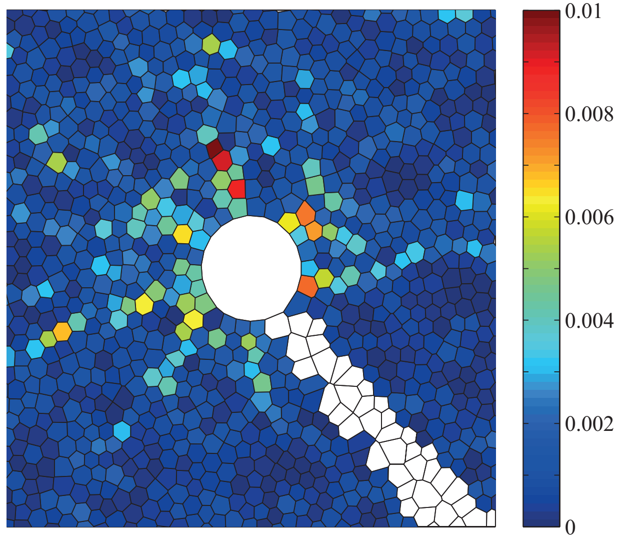

$\gamma = J_2(\varepsilon)$



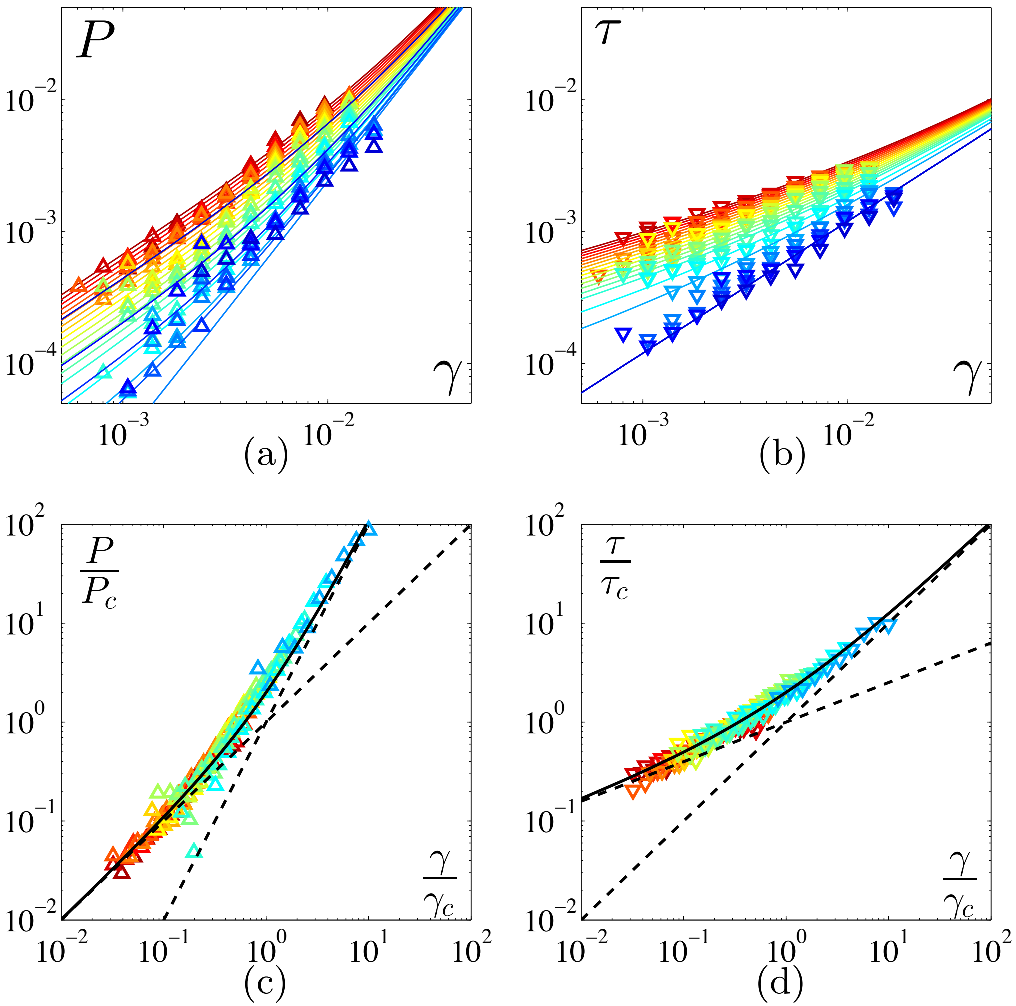

$P = Tr(\sigma)$




$\tau = J_2(\sigma)$



# Parametric plot of stress vs strain




$$P = \left[R_0 + R_{nl}(\Delta\phi,\gamma)\right]\gamma^2$$
$$\tau = 2\left[G_0 + G_{nl}(\Delta\phi,\gamma)\right]\gamma$$

$$R_{nl}(\Delta\phi,\gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ a\Delta\phi^{\mu}\gamma^{\alpha-2} & \text{for } \phi > \phi_J \end{cases}$$

$$G_{nl}(\Delta\phi,\gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ b\Delta\phi^{\nu}\gamma^{\beta-1} & \text{for } \phi > \phi_J \end{cases};$$

- $\mu = 1.7$ $\alpha = 1.0$
- $\nu = 1.0$ $\beta = 0.4$
- => $\gamma_c \sim \Delta\phi^{\zeta}$; $\zeta=1.7$

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# Conclusion

- Vibrated granular media are suitable tools for probing the vicinity of jamming, (in particular low enough T_eff)
- Two distinct crossovers (one dynamical, one structural) converge toward J-point in the limit of low vibration
- Pulling an intruder in vibrated hard discs has allowed us to probe the yield stress of “thermal origin” and reveals complex pinning – depinning like dynamics
- Inflating an intruder in soft photo-elastic discs => Non linear rheology
- Thank you!

Further readings :


- **Europhysics Letters, 83, 46003, (2008).**
- **Soft Matter, 6 (13), 3059–3064, (2010).**
- **Phys Rev Lett 103 12800 (2009).**
- **Europhysics Letters, 100, 44005 (2012).**
- **Soft Matter (2013) to appear.**