Microstructure of a plastic flow before failure

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Plasticity and failure of amorphous material

Amorphous and athermal material: Foam, granular material, metallic glasses, ...

Deformations of amorphous and athermal material:

Strain localization

M.W.Chen (2008)

Desrues et al (2002)

H.Herrman
Plasticity to localization

Falk (1998)

Plastic properties of metallic glasses may be understood with the hypothesis of localized plastic events [Argon (1979)]

-Localized plastic events
-Elastic Coupling

Strain localization

Dense colloids
Schall, Weitz, Spaepen (2007)

D.Rodney et al. (2009)
State of the art

- Many numerical/theoretical studies
- Experiments: localized events,
  - very few clear « evidence » of coupling

Visualization of coupling?
Link between elastic events and band formation?

Outline

- Introduction
- Experimental setup
  - Mechanical part
  - Detection of fluctuations of deformation
- A typical experiment
- Permanent shear band (the end)
- Transient structure (the beginning)
  - Structure
  - Micromechanical model
- From transient to permanent shear band?
Experimental setup: mechanical setup

Biaxial stress test:
- Well described in geomechanics
- Homogeneous applied stress
- Elongational flow of hard spheres

Face view
Side view

- Material: glass beads d=100µm
- Confining pressure + applied pressure
- Relative deformation $-\Delta L/L = \varepsilon$
- Compression at $d\varepsilon/dt \sim 10^{-5}$ s$^{-1}$
- No cohesion, no grain breaking,...

A. Le Bouil et al., Gran. Mat. 16 1 (2014)
Experimental setup: Maps of deformations

Correlation maps between two successive images:
\[ g_1 = \frac{\langle I_1, I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 \]

Color code:
- White: \( g_1 = 1 \)  
  local deformation \( \varepsilon < 10^{-6} \)
- Red: \( g_1 = 0 \)  
  local deformation \( \varepsilon > 10^{-4} \)

- Only volume near surface (depth few d) is probed
- Very sensitive method (deformation \( \sim 10^{-6} - 10^{-4} \), ...)

M. Erpelding PhD thesis
M. Erpelding et al, PRE 78, 046104 (2008)
A typical experiment
**Permanent shear band (the end) - Mohr-Coulomb analysis**

Desrues et al. (2002)

Solid friction:
- dry, no cohesion
- no grains breakage
- no grains deformation ($\sigma \ll E$)

$$|\tau| \geq \mu |\sigma|$$

$$\theta_{MC} = 45^\circ + \frac{1}{2} \arcsin \left[ \frac{\sigma_{yy} - \sigma_{xx}}{\sigma_{yy} + \sigma_{xx}} \right]$$

($64^\circ$)  ($62^\circ$)
Plastic flow before failure: transient

The small scale micro structure of flow is visible only at strain increment less than $\sim 10^{-4}$. "fluctuation of the plastic flow"
Plastic flow before failure: structure

Snapshot of the plastic flow: \( g_I(\varepsilon, r) \)

Spatial correlation function of plastic flow
\[
\psi(\varepsilon, r) = \langle g_I(\varepsilon, r') g_I(\varepsilon, r + r') \rangle - \langle g_I(\varepsilon, r') \rangle \langle g_I(\varepsilon, r + r') \rangle
\]

- The plastic flow is structured
- Directionality \( \theta_\varepsilon = +/- 50-54^\circ \)
- \( \psi(\varepsilon, r) \) increases as loading progresses
- Very different from rupture (permanent, \( \theta_{MC} = +/- 64^\circ \))
Micro-structure of the plastic flow

Hypothesis: localized plastic event

Model: material treated as a continuous isotropic elastic matrix

Deformation tensor $e^*$

Stress tensor $\tilde{\sigma} = S \varepsilon^*$

Here 2D plane strain (very similar in 3D) with:

$$\sigma_{xx} + \tilde{\sigma}_{xx} - (\sigma_{yy} + \tilde{\sigma}_{yy}) = \sigma_{xx} - \sigma_{yy} + C f(\theta)$$

Total stress difference

Applied stress difference

Additional stress due to the reorganization

http://micro.stanford.edu/~caiwei/me340b/content/me340b-notes_v01.pdf
Micro-structure of the plastic flow

\[ \sigma_{xx} + \tilde{\sigma}_{xx} - (\sigma_{yy} + \tilde{\sigma}_{yy}) = \sigma_{xx} - \sigma_{yy} + C f(\theta) \]

\[ f(\theta) = (e_{xx}^* - e_{yy}^*) \left[ -\frac{15}{4} \cos(4\theta) + \frac{8\nu-7}{4} \right] - \frac{3}{2} (e_{xx}^* + e_{yy}^*) \cos(2\theta) \]

\( f(\theta) > 0 \): additional stress adds to the applied stress

Dilatant ? : \( e_{xx}^* > 0 \quad e_{yy}^* < 0 \quad e_{xx}^* + e_{yy}^* > 0 \)

Maximum of \( f(\theta) \) for angle \( \theta_E^* \):

\[ \cos(2\theta_E^*) = \frac{3}{10} \frac{e_{yy}^* + e_{xx}^*}{e_{yy}^* - e_{xx}^*} \]

\( e_{xx}^* + e_{yy}^* = 0 \quad \theta_E^* = \pm 45^\circ ; \pm 135^\circ \)

\( |e_{xx}^*| >> |e_{yy}^*| \quad \theta_E^* \approx \pm 54^\circ ; \pm 126^\circ \)
Micro-structure of the plastic flow

Spatial correlation function of plastic flow

\[ \psi(\varepsilon, \mathbf{r}) = \langle g_I(\varepsilon, \mathbf{r}') g_I(\varepsilon, \mathbf{r}+\mathbf{r}') \rangle - \langle g_I(\varepsilon, \mathbf{r}') \rangle \langle g_I(\varepsilon, \mathbf{r}+\mathbf{r}') \rangle \]

Confirmed by numerical Discrete Element Method simulations

\[ \cos(2\theta^*_E) = \frac{3}{10} \frac{e^*_{yy} + e^*_{xx}}{e^*_{yy} - e^*_{xx}} \]

\[ |e^*_{xx}| > > |e^*_{yy}| \]

\[ e^*_{xx} + e^*_{yy} = 0 \]

N.Guo & J.Zhao, PRE 89, 042208 (2014)
From isolated events to shear band

Spatial correlation function of plastic flow
\[ \psi(\varepsilon, r) = \langle g_1(\varepsilon, r') \, g_1(\varepsilon, r+r') \rangle - \langle g_1(\varepsilon, r') \rangle \langle g_1(\varepsilon, r+r') \rangle \]

Coupling along +/- \( \theta_E \) increases as system approaches failure

Anisotropy characterization:
\[ \chi(\varepsilon, r) = \frac{1}{2} [\Psi(\varepsilon, r, \theta_E) + \Psi(\varepsilon, r, -\theta_E)] - \Psi_{iso}(\varepsilon, r) \]
From isolated events to shear band

The locus of rearrangement shows some spatial correlation from one event to the next, but these correlations decay quickly after just a few plastic events (...). We observe no evidence for the kinds of pronounced persistent shear localization which is seen in many experiments and simulations (…). We find it likely that the persistent localization observed elsewhere is due largely to effects of the boundary.

- **Vandembroucq et al. PRB (2011)**
- **Maloney & Lemaître. PRE (2006)**
Conclusion

- A plastic flow of hard spheres
- The plastic flow is structured
- Transient & oriented structure
- Agreement with Eshelby-like structure
- Increase of elastic coupling

- Only indirect link with the failure

References:

- A. Le Bouil et al., Gran. Mat. 16 1 (2014) (experimental setup)

see also:

- M. Erpelding et al, PRE 78, 046104 (2008) (spatially resolved DWS setup)
- A. Amon et al, PRE 87, 012204 (2013) (Failures in inclined plane)

http://perso.univ-rennes1.fr/jerome.crassous/