# Microstructure of a plastic flow before failure

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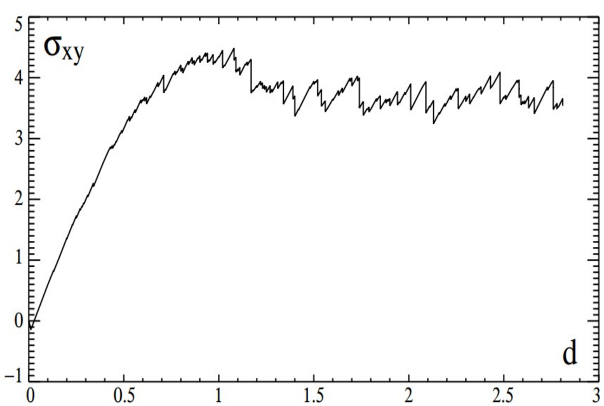
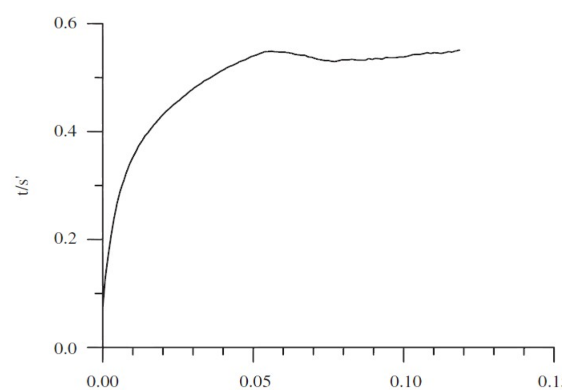
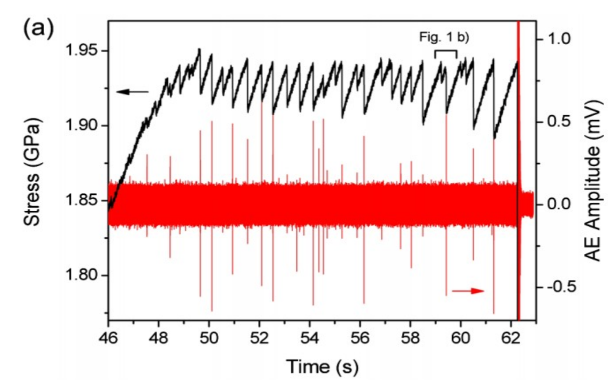
# Plasticity and failure of amorphous material

Amorphous and athermal material : Foam, granular material, metallic glasses, ...

Deformations of amorphous and athermal material :



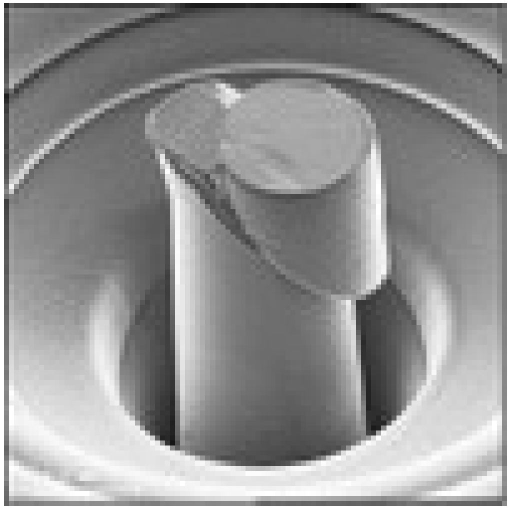




Kabla (2003)

Desrue et al. (2002)

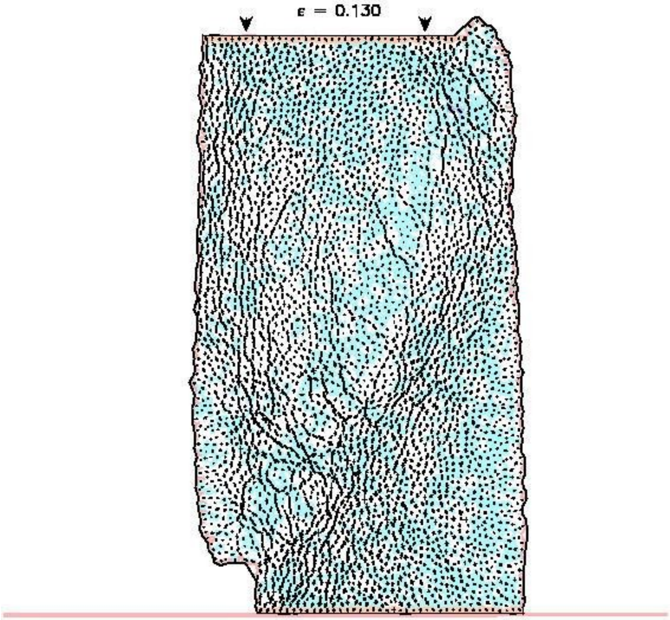
Klaumünzer et al (2011)

# Strain localization



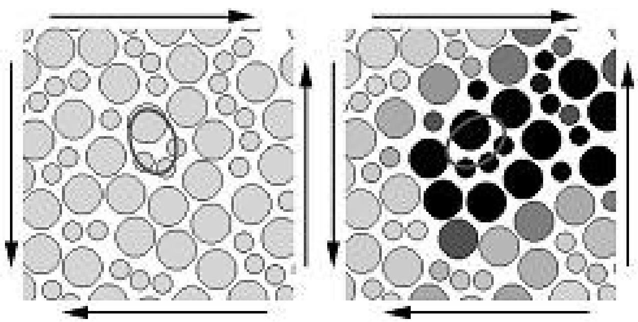
M.W.Chen (2008)

Desrues et al (2002)



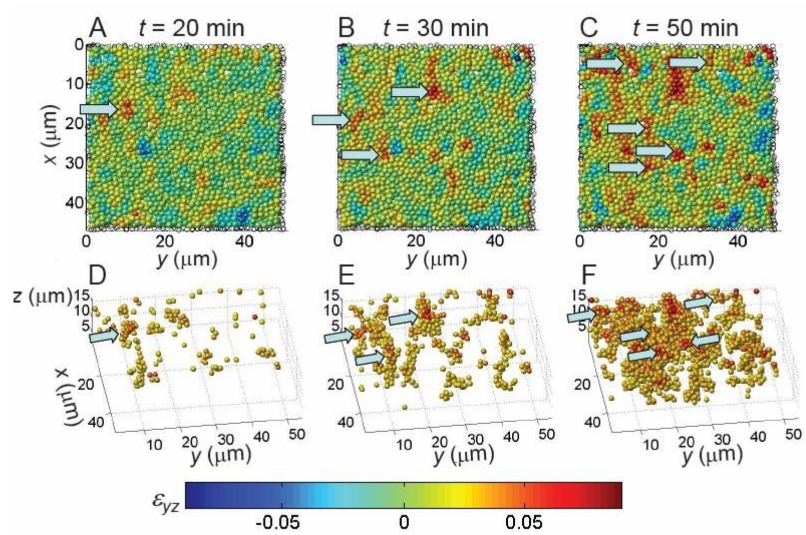

H.Herrman

# Plasticity to localization



Falk (1998)

Plastic properties of metallic glasses may be understood with the hypothesis of localized plastic events [Argon (1979)]



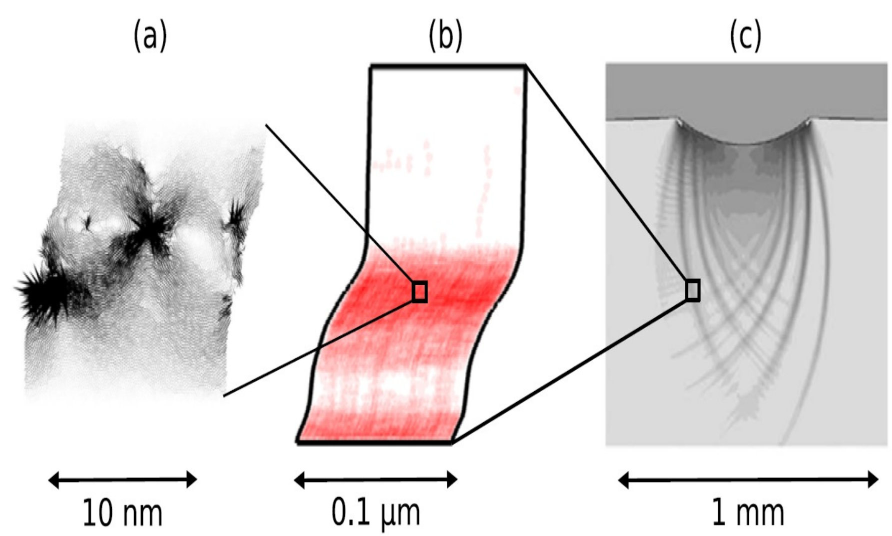

Dense colloids

*Schall, Weitz, Spaepen (2007)*

-localized plastic events

-Elastic Coupling

→ Strain localization




D.Rodney et al. (2009)

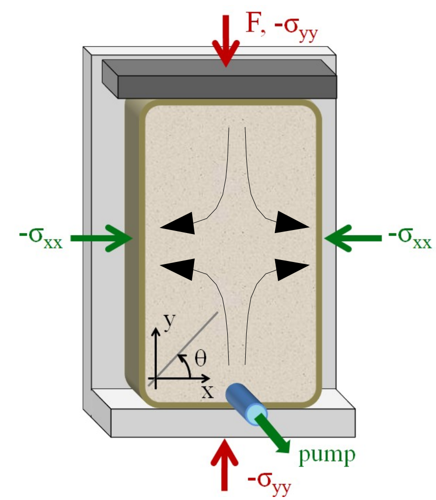
# State of the art

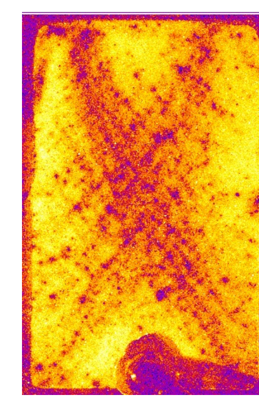
- Many mumerical/theoritical studies
- Experiments : -localized events,
  - very few clear « evidence » of coupling

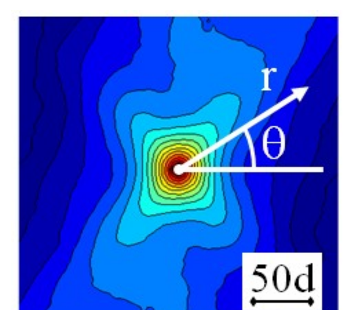
- Vizualization of coupling ?
- Link between elastic events and band formation ?

# Outline

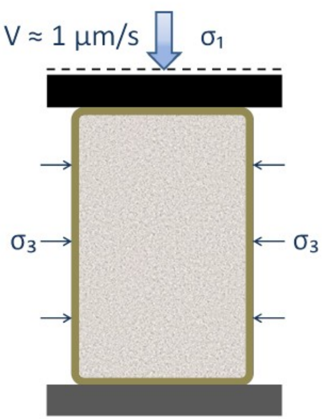
- Introduction
- Experimental setup
  - -Mechanical part
  - -Detection of fluctuations of deformation
- A typical experiment
- Permanent shear band (the end)
- Transient structure (the beginning)
  - -Structure
  - -Micromechanical model
- From transient to permanent shear band ?



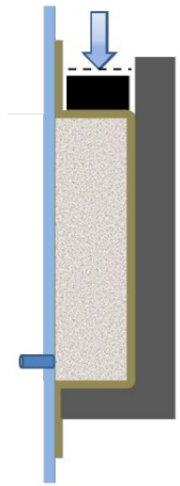





# Experimental setup : mechanical setup



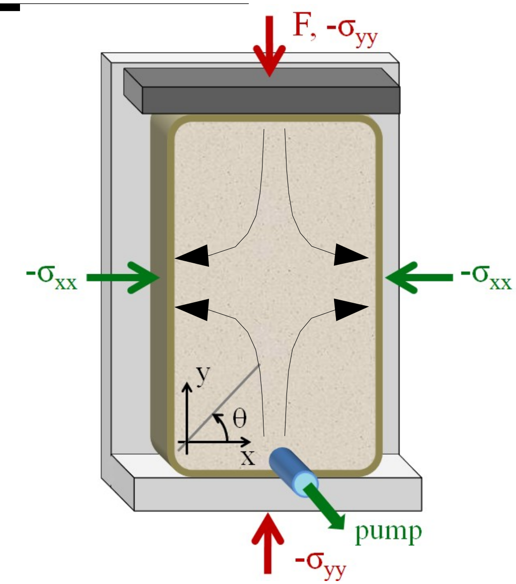

Face view

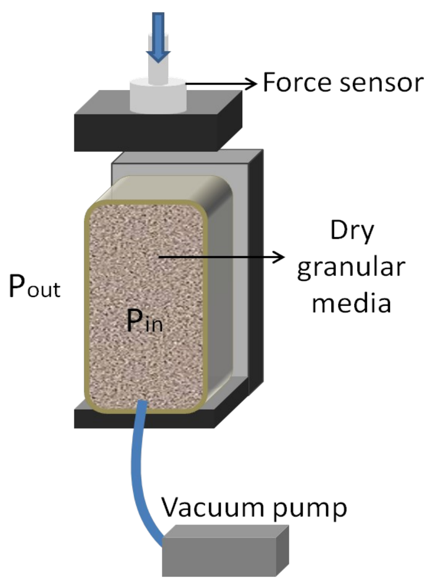


Side view

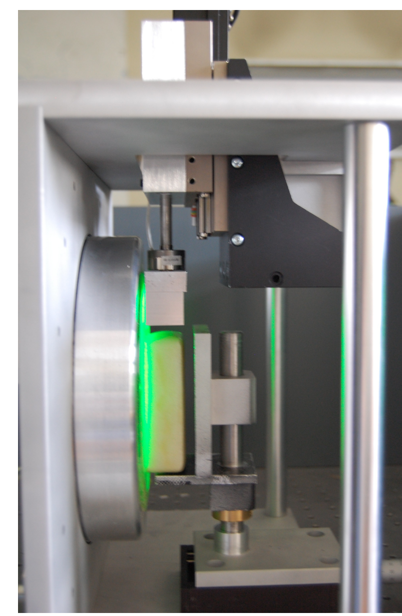
Biaxial stress test:

- Well described in geomechanics
- Homogeneous applied stress
- Elongational flow of hard spheres



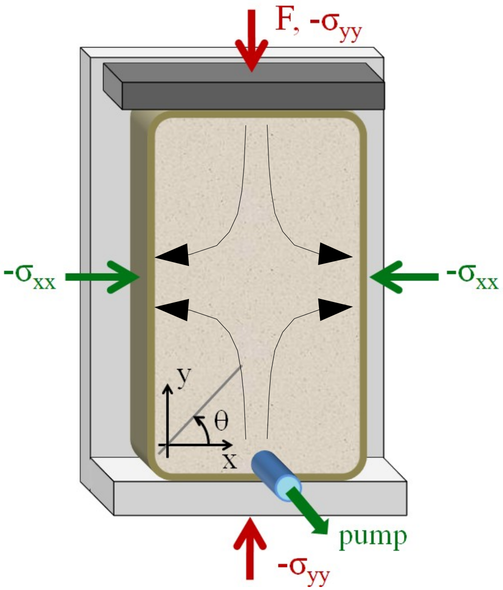



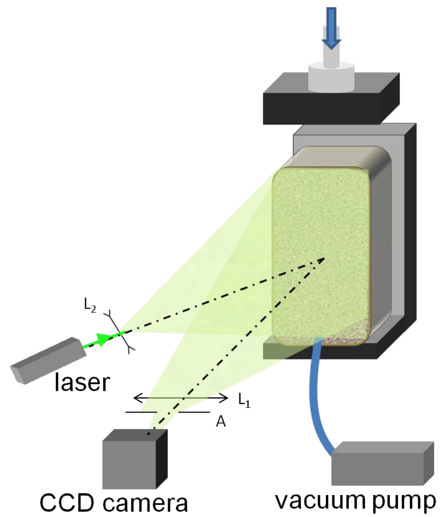

- Material: glass beads d=100µm
- Confining pressure + applied pressure
- Relative deformation $-\Delta L/L = \varepsilon$
- Compression at $d\varepsilon/dt \sim 10^{-5}\,s^{-1}$
- No cohesion, no grain breaking,...

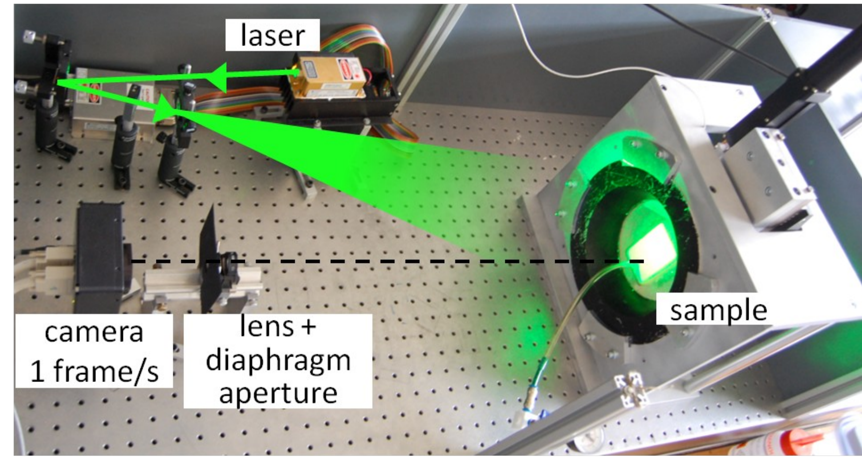


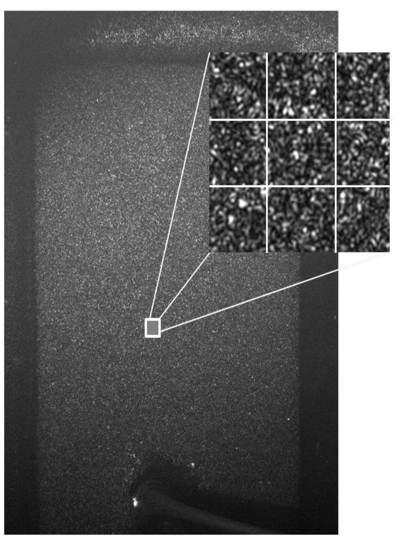
A.Le Bouil et al., Gran.Mat. **16** 1 (2014)

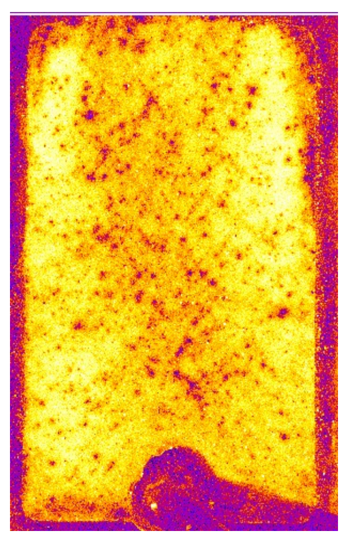
# Experimental setup : Maps of deformations





Correlation maps between two successive images

$$g_I = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle - 1$$

Color code:

-White: $g_I = 1$

local deformation $\varepsilon < 10^{-6}$

-Red

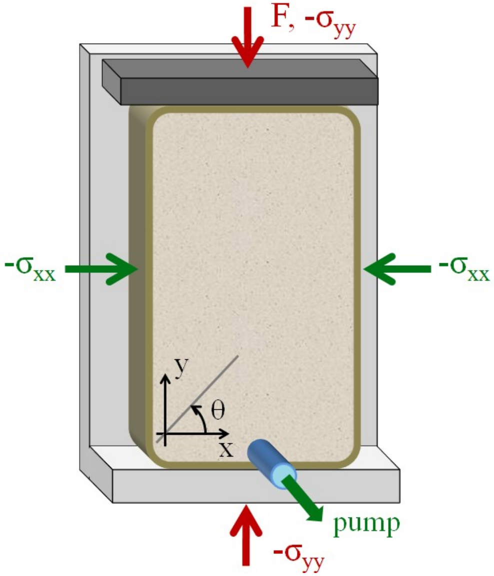
$g_I = 0$

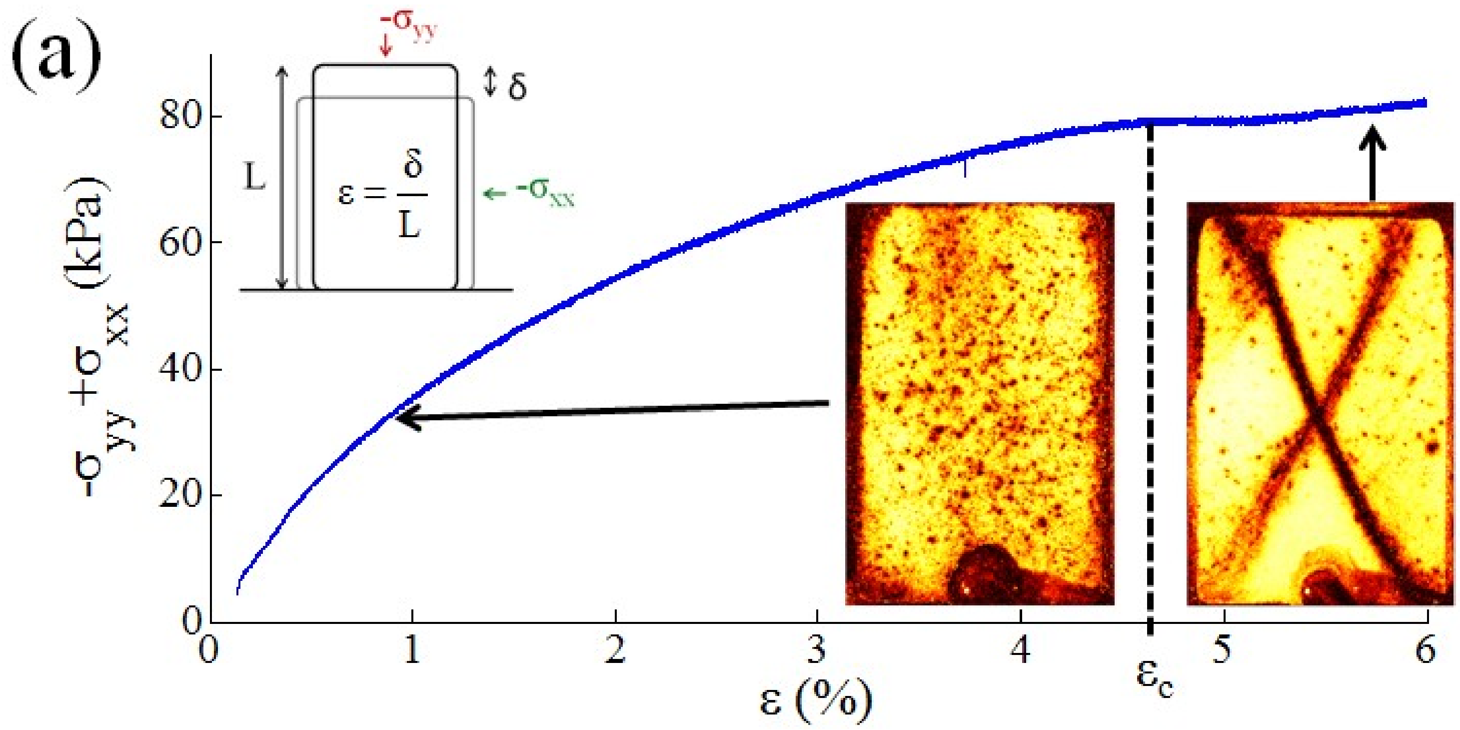
local deformation $\varepsilon > 10^{-4}$

- M.Erpelding PhD thesis
- M.Erpelding et al, PRE 78, 046104 (2008)

- Only volume near surface (depth few d) is probed
- Very sensitive method (deformation ~ $10^{-6}$ - $10^{-4}$, ... )

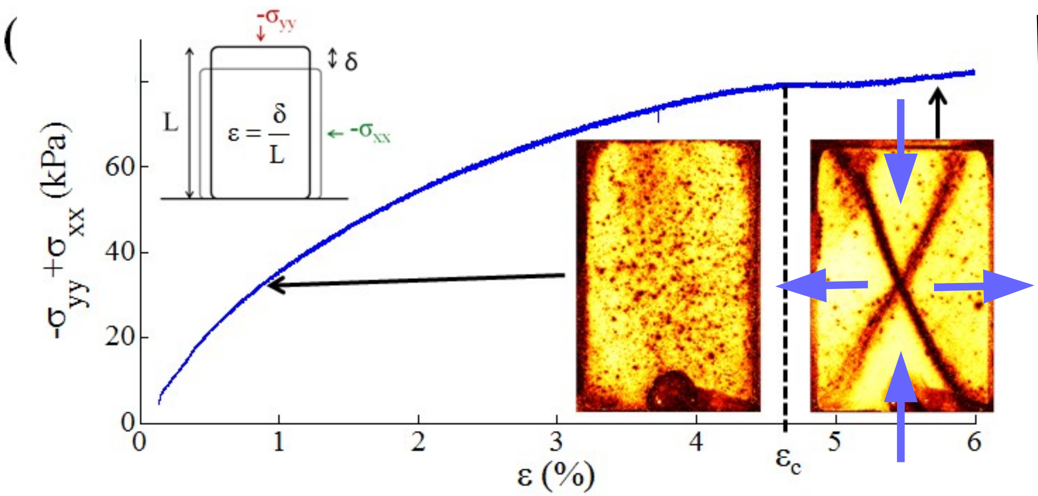
# A typical experiment

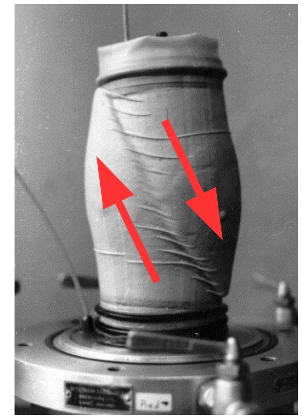


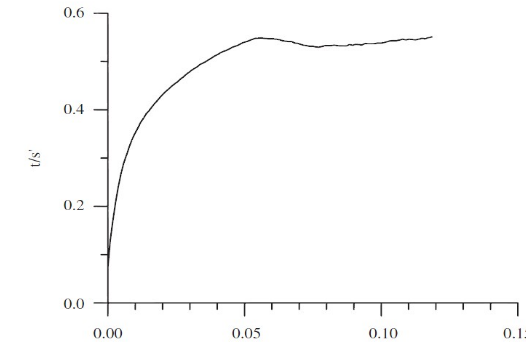




Stress-deformation curve

# Permanent shear band (the end) - Mohr-Coulomb analysis



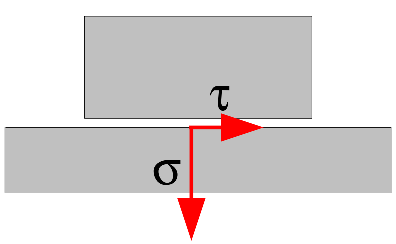





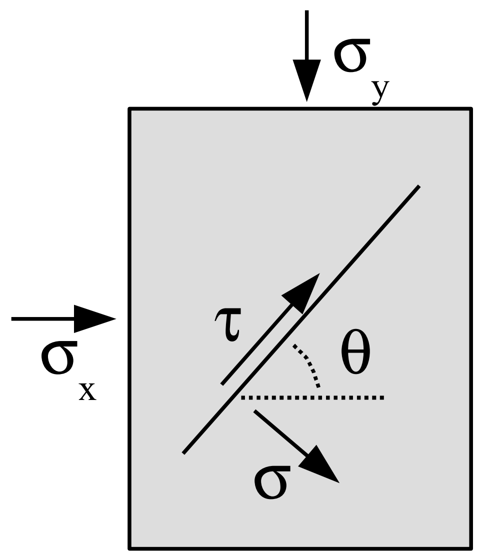

Desrues et al. (2002)

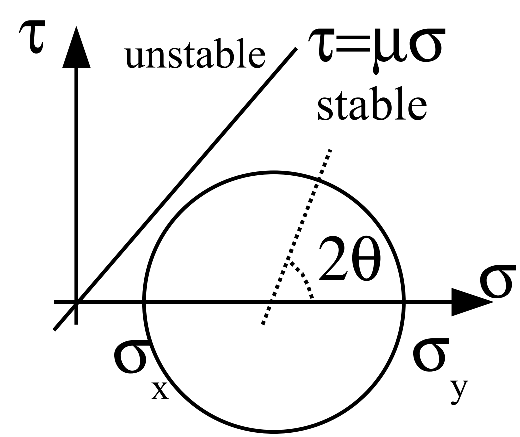
Solid friction :

- dry, no cohesion
- no grains breakage
- no grains deformation (σ << E)




$$|\tau| \geq \mu|\sigma|$$

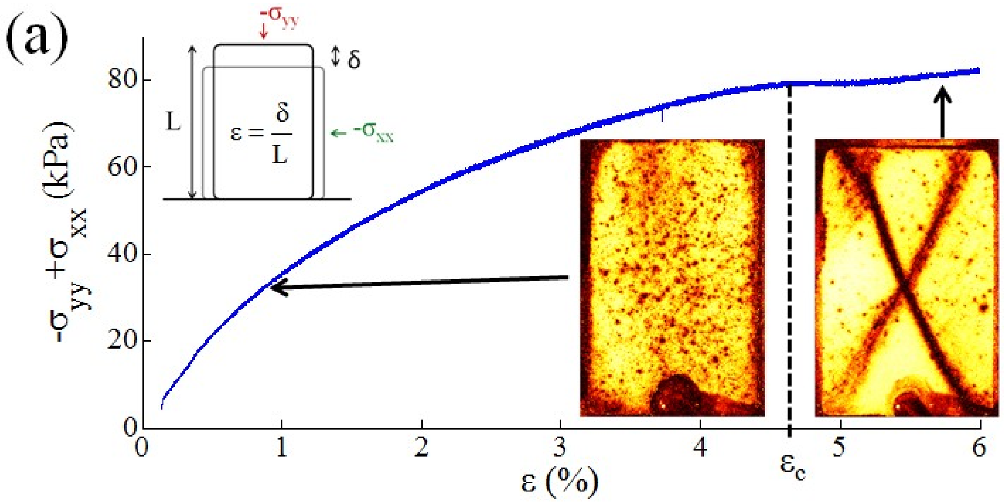


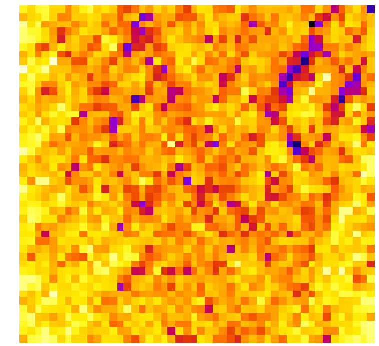




$$\theta_{MC} = 45^\circ + \frac{1}{2} asin\left[\frac{\sigma_{yy}-\sigma_{xx}}{\sigma_{yy}+\sigma_{xx}}\right]$$

(64°) (62°)

# Plastic flow before failure: transient



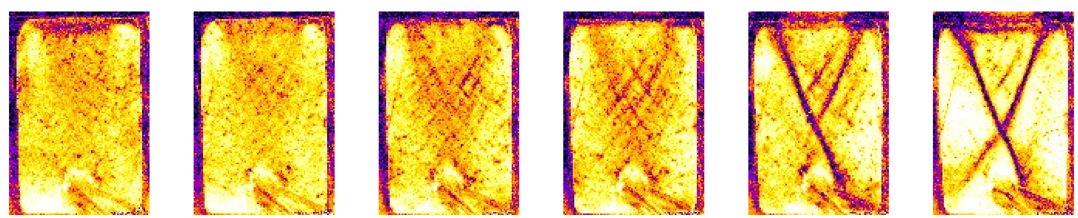



$\varepsilon$

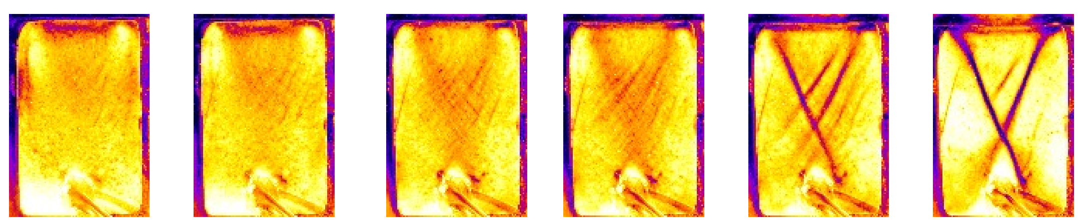
Loading

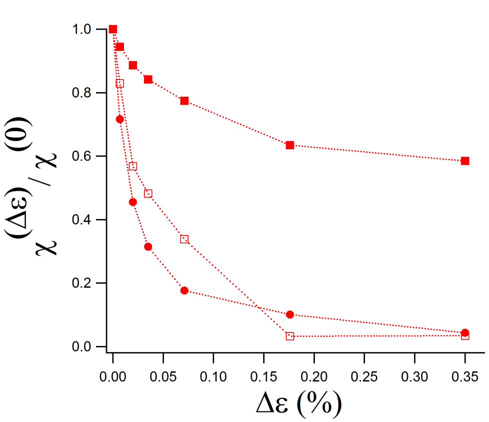
$\delta\varepsilon = 3x10^{-5}$

1 image



100 images



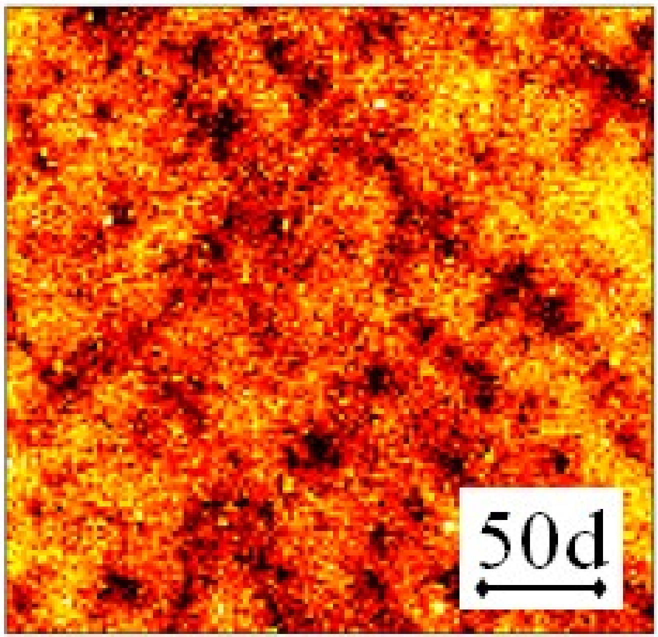



The small scale micro structure of flow is visible only at strain increment less than $\sim 10^{-4}$

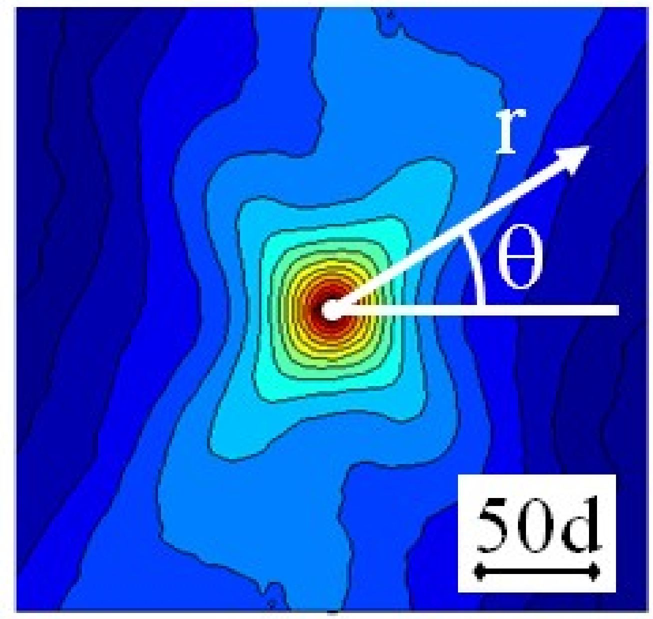
“fluctuation of the plastic flow”

# Plastic flow before failure: structure

Snapshot of the plastic flow: $g_I(\varepsilon,r)$

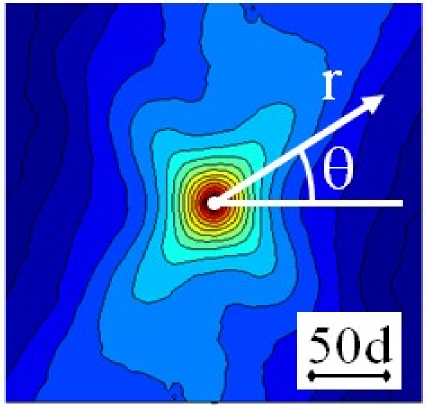


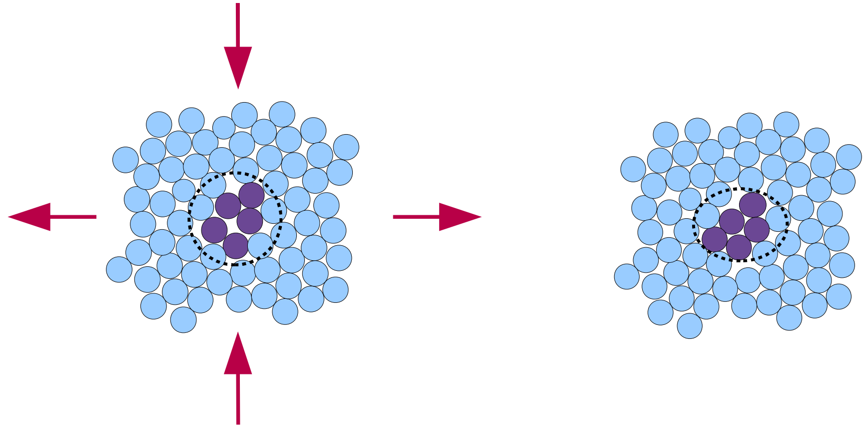

Spatial correlation function of plastic flow
$\psi(\varepsilon,\mathbf{r}) = \langle g_I(\varepsilon,\mathbf{r}')\, g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle - \langle g_I(\varepsilon,\mathbf{r}')\rangle\langle g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle$




-The plastic flow is structured
- Directionality $\theta_E$=+/- 50-54°
- $\psi(\varepsilon,r)$ increases as loading progresses
- Very different from rupture (permanent, $\theta_{MC}$=+/- 64°)

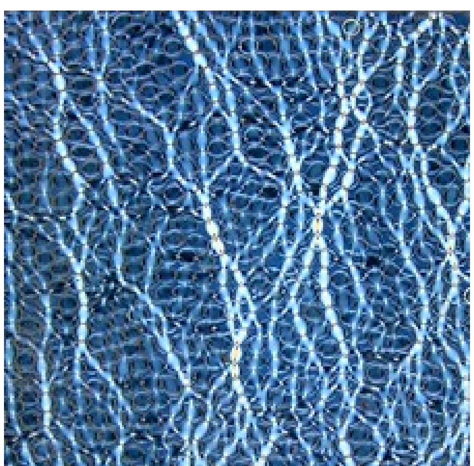
# Micro-structure of the plastic flow



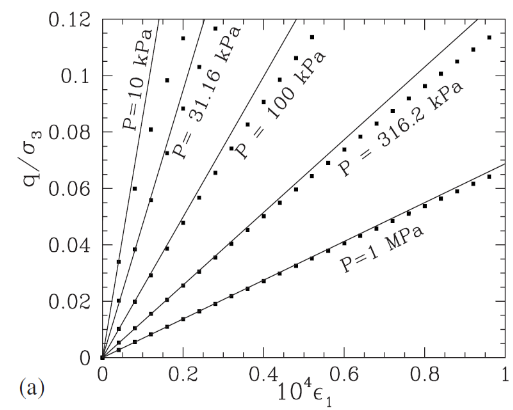



Hypothesis : localized plastic event

Model : material treated as a <u>continuous</u> <u>isotropic</u> <u>elastic</u> matrix

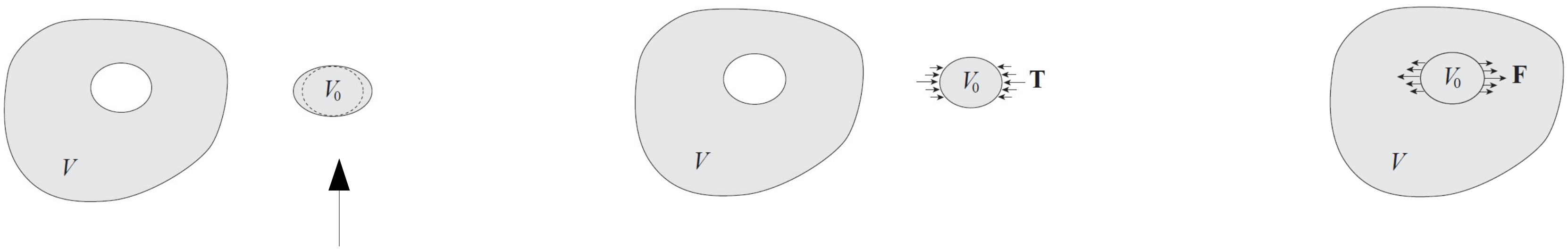


B.Behringer




I.Agnolin and J.-N. Roux, PRE (2004)

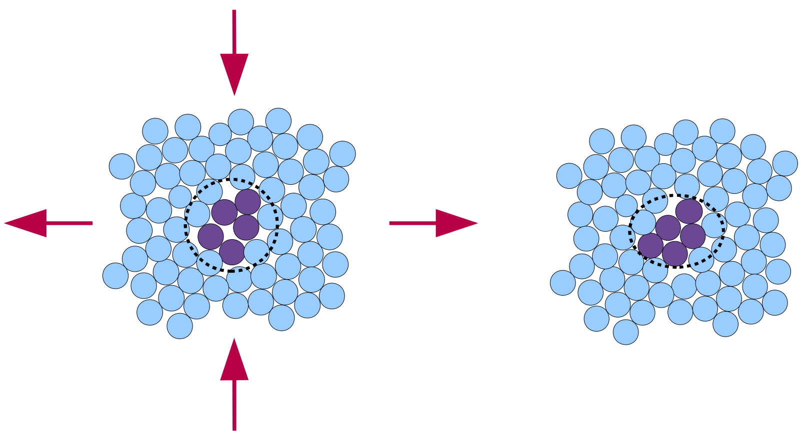
# Eshelby inclusion problem




Deformation tensor e*

Stress tensor $\tilde{\sigma} = S\,\varepsilon*$

http://micro.stanford.edu/~caiwei/me340b/content/me340b-notes_v01.pdf



Here 2D plane strain (very similar in 3D) with:

$$e^* = \begin{pmatrix} e^*_{xx} & 0 \\ 0 & e^*_{yy} \end{pmatrix}$$

$$\tilde{\sigma}_{xx} - \tilde{\sigma}_{yy} \propto f(\theta)$$

with : $f(\theta) = (e^*_{xx} - e^*_{yy})\left[-\frac{15}{4}\cos(4\theta) + \frac{8\nu-7}{4}\right] - \frac{9}{2}(e^*_{xx} + e^*_{yy})\cos(2\theta)$

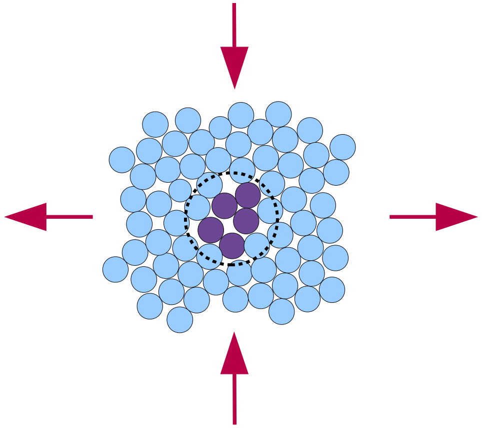
$$\sigma_{xx} + \tilde{\sigma}_{xx} - (\sigma_{yy} + \tilde{\sigma}_{yy}) = \sigma_{xx} - \sigma_{yy} + C\,f(\theta)$$

Total stress difference

Applied stress difference

Additional stress due to the reorganization

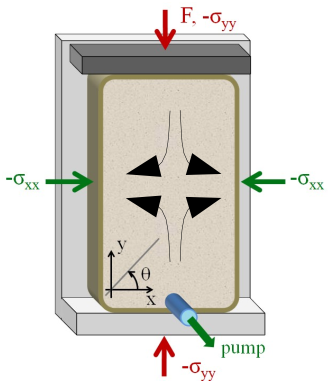
# Micro-structure of the plastic flow

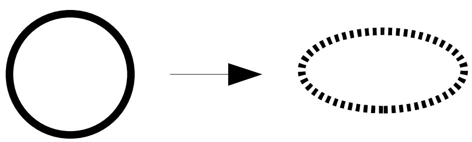


$$\sigma_{xx} + \tilde{\sigma}_{xx} - (\sigma_{yy} + \tilde{\sigma}_{yy}) = \sigma_{xx} - \sigma_{yy} + C f(\theta)$$

$$f(\theta) = (e^*_{xx} - e^*_{yy})\left[-\frac{15}{4}\cos(4\theta) + \frac{8\nu-7}{4}\right] - \frac{9}{2}(e^*_{xx} + e^*_{yy})\cos(2\theta)$$

$f(\theta) > 0$ : additionnal stress adds to the applied stress

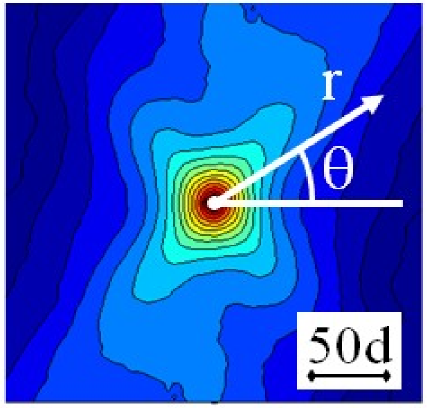


$$e^*_{xx} > 0$$

$$e^*_{yy} < 0$$

Dilatant ? : $e^*_{xx} + e^*_{yy} > 0$




Maximum of $f(\theta)$ for angle $\theta^*_E$ : $\cos(2\theta^*_E) = \frac{3}{10}\frac{e^*_{yy} + e^*_{xx}}{e^*_{yy} - e^*_{xx}}$

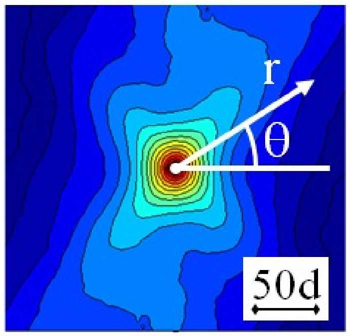
$e^*_{xx} + e^*_{yy} = 0$ $\quad\quad \theta^*_E = \pm 45°; \pm 135°$

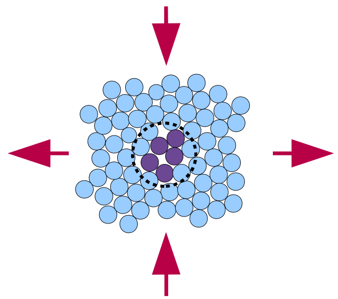
$|e^*_{xx}| >> |e^*_{yy}|$ $\quad\quad \theta^*_E \approx \pm 54°; \pm 126°$

# Micro-structure of the plastic flow

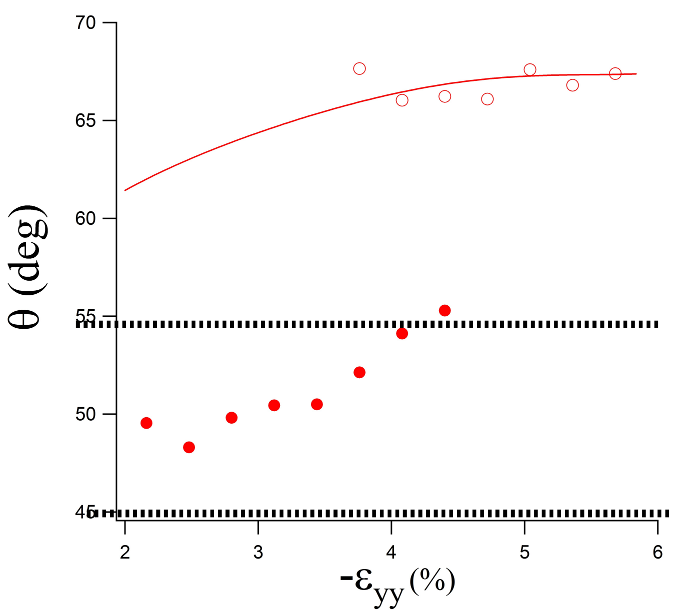
Spatial correlation function of plastic flow

$\psi(\varepsilon,\mathbf{r}) = \langle g_I(\varepsilon,\mathbf{r}')\, g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle - \langle g_I(\varepsilon,\mathbf{r}')\rangle \langle g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle$



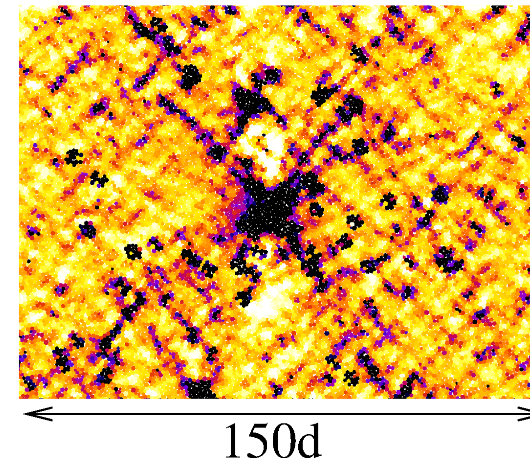



$$\cos(2\theta_E^*) = \frac{3}{10}\,\frac{e_{yy}^* + e_{xx}^*}{e_{yy}^* - e_{xx}^*}$$

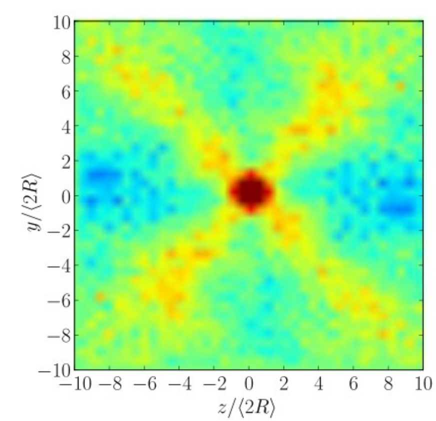



$$|e_{xx}^*| >> |e_{yy}^*|$$

$$e_{xx}^* + e_{yy}^* = 0$$

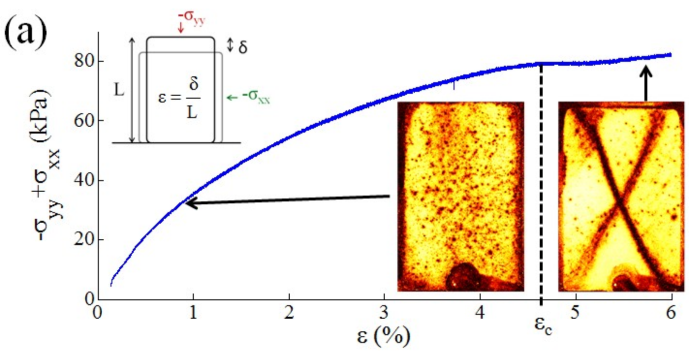



Confirmed by numerical Discrete Element Method simulations



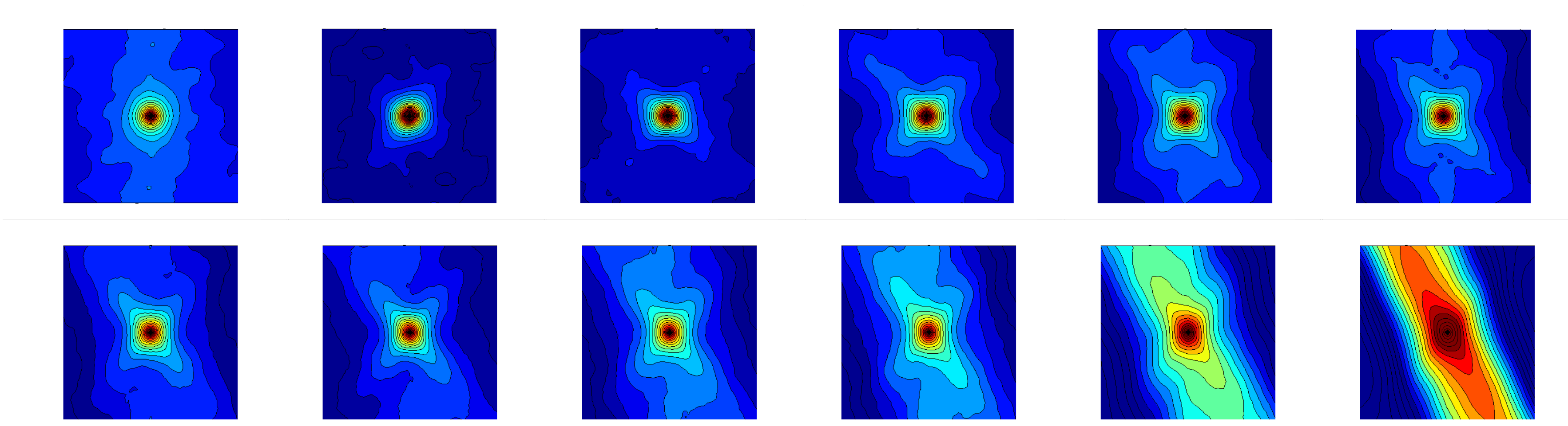

N.Guo & J.Zhao, PRE 89, 042208 (2014)

# From isolated events to shear band




Spatial correlation function of plastic flow

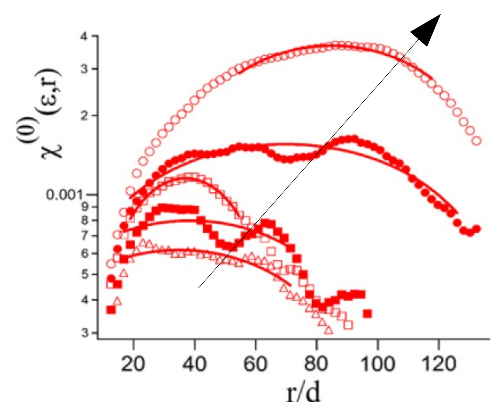
$\psi(\varepsilon,\mathbf{r}) = \langle g_I(\varepsilon,\mathbf{r}')\, g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle - \langle g_I(\varepsilon,\mathbf{r}')\rangle\langle g_I(\varepsilon,\mathbf{r}+\mathbf{r}')\rangle$



Coupling along +/- $\theta_E$ increases as system approaches failure

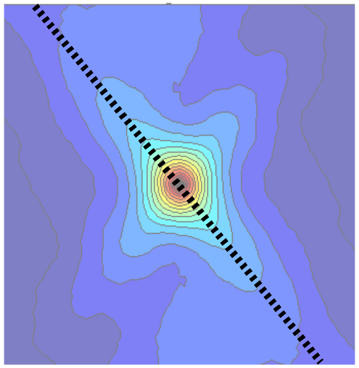
Anisotropy characterization:

$$\chi(\epsilon, r) = \frac{1}{2}[\Psi(\epsilon, r, \theta_E) + \Psi(\epsilon, r, -\theta_E)] - \Psi_{iso}(\epsilon, r)$$

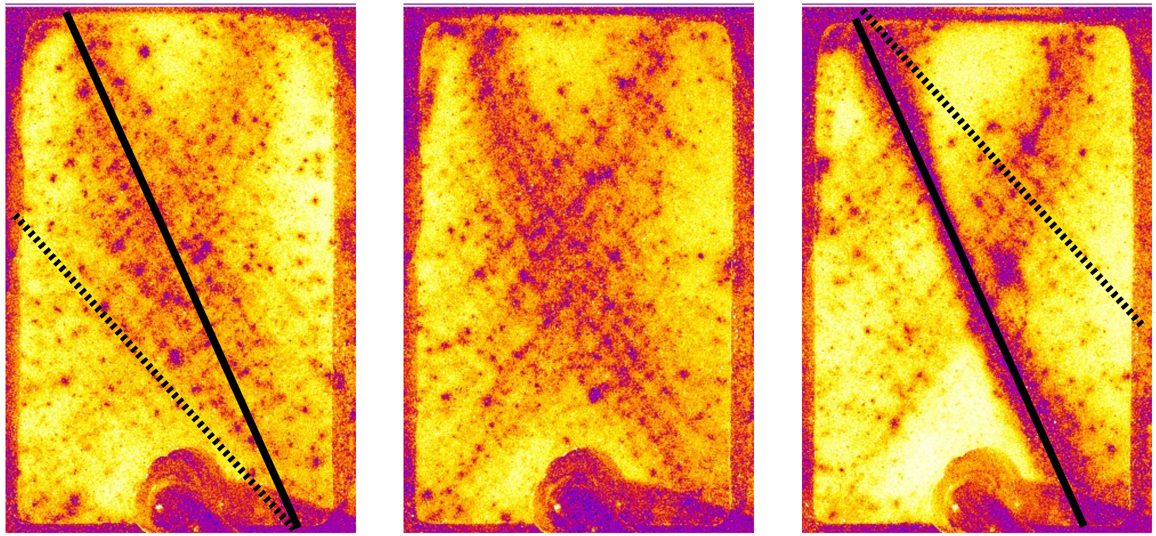


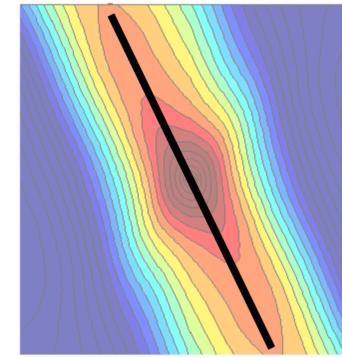

increase of loading

# From isolated events to shear band



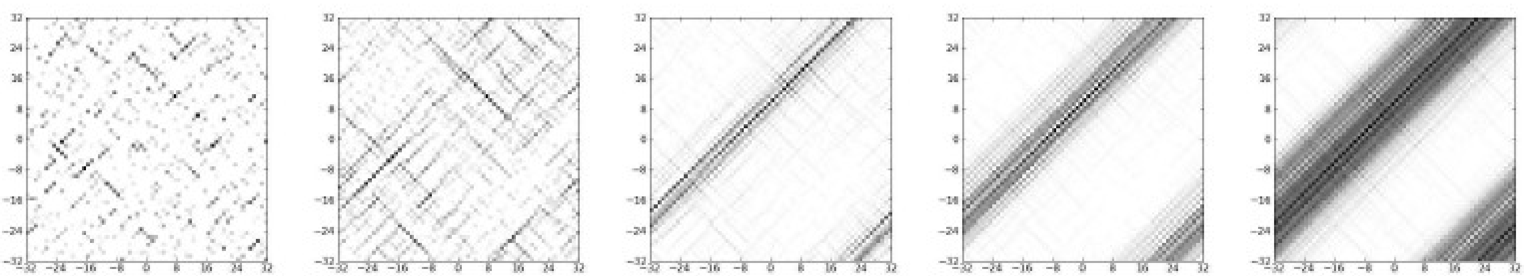
Eshelby $\theta_E$





Mohr-Coulomb $\theta_{MC}$

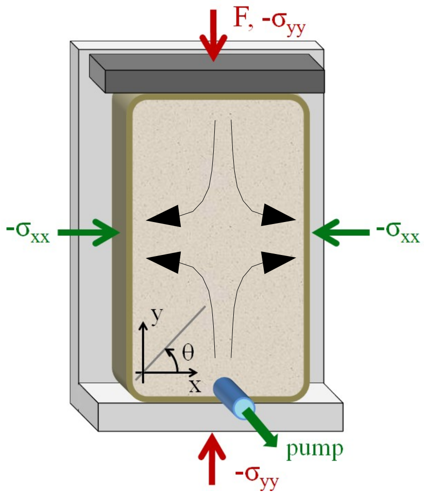
- Vandembroucq et al. **PRB** (2011)



The locus of rearrangement shows some spatial correlation from one event to the next, **but these correlations decay quickly after just a few plastic events** (...). We observe no evidence for the kinds of pronounced persistent shear localization which is seen in many experiments and simulations (…)
we find it likely that **the persistent localization observed elsewhere is due largely to effects of the boundary.**

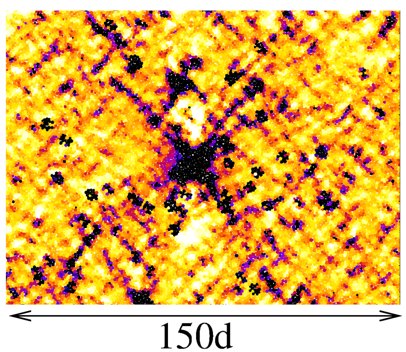
- Maloney & Lemaître. **PRE** (2006)

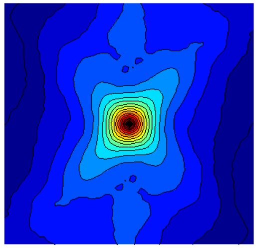
# Conclusion

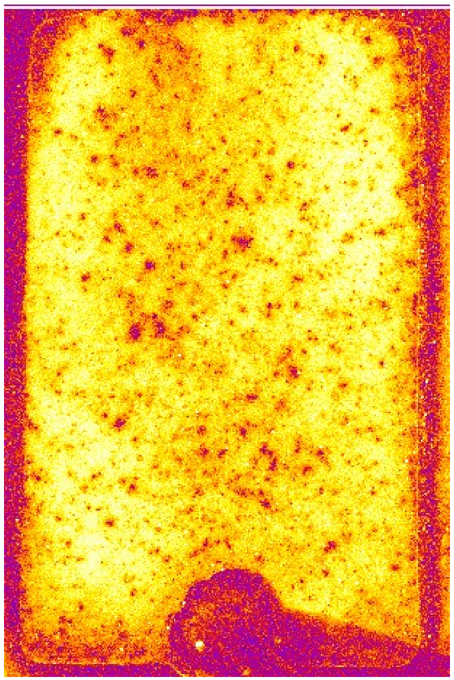



- A plastic flow of hard spheres
- The plastic flow is structured
- Transient & oriented structure
- Agreement with Eshelby-like structure
- Increase of elastic coupling

- Only indirect link with the failure





References :
- A.Le Bouil et al., Gran.Mat. **16** 1 (2014) (experimental setup)
- A.Le Bouil et al.,arXiv - PRL, in press (2014) (structure of plastic flow)

see also :
- M.Erpelding et al, PRE **78**, 046104 (2008) (spatialy resolved DWS setup)
- A.Amon et al, PRL 108, 135502 (2012) (A creep flow)
- A.Amon et al, PRE 87, 012204 (2013) (Failures in inclined plane)

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