From shear transformations to depinning transitions

Lattice models for amorphous plasticity

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Phys. Rev. E 88, 062403 (2013)

Outline

Lattice-based depinning models for amorphous materials

→ Yield as a depinning phase transition

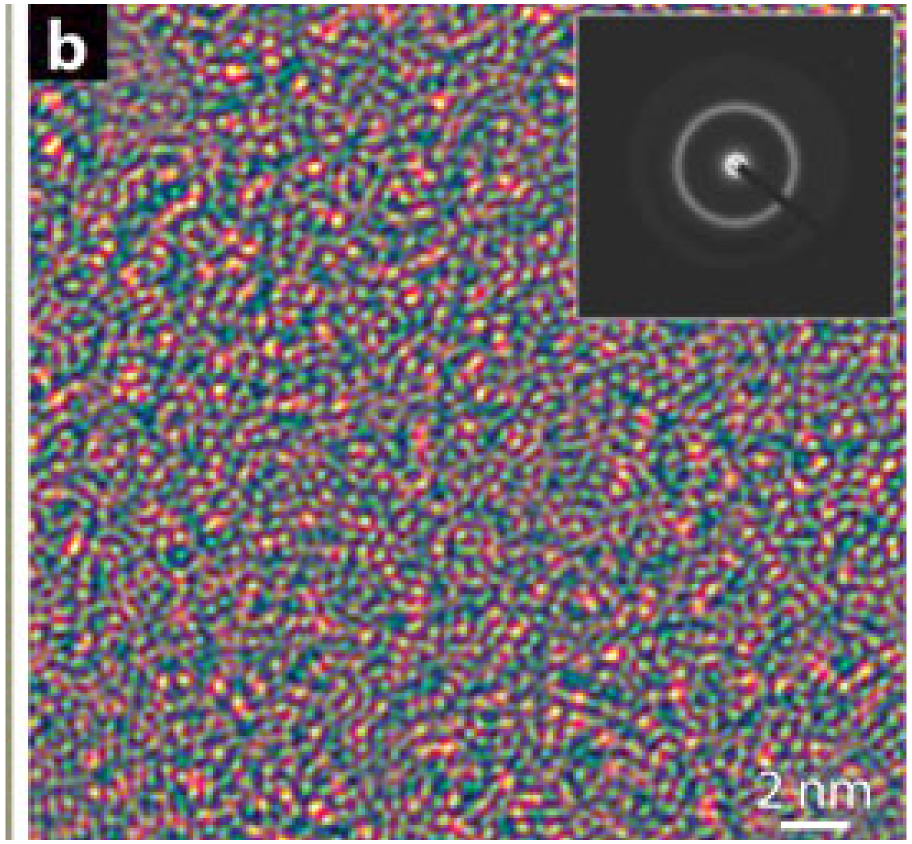
→ Universality class?

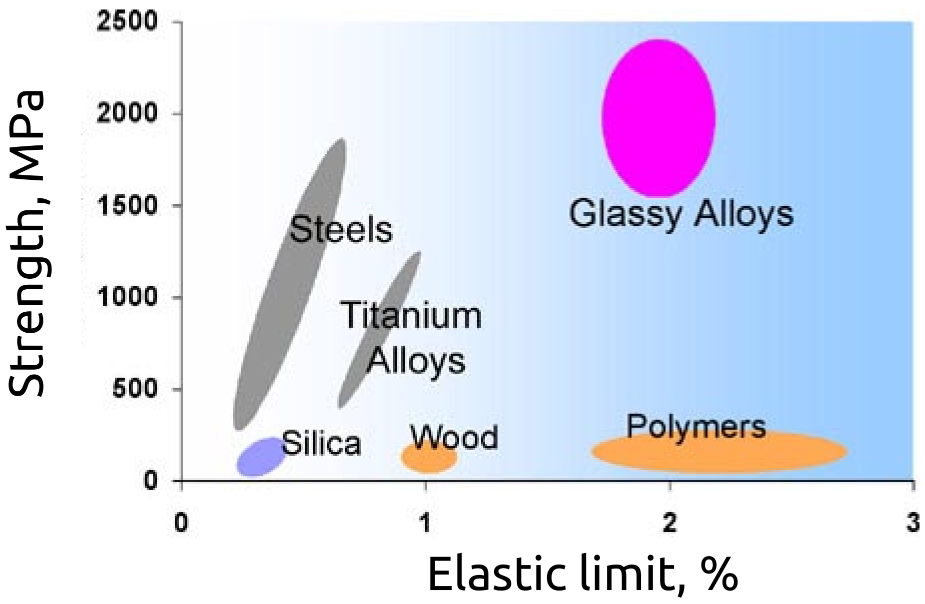
→ Nonuniversal localization effects

Verifying and moving beyond scalar models

Amorphous materials

Bulk metallic glasses



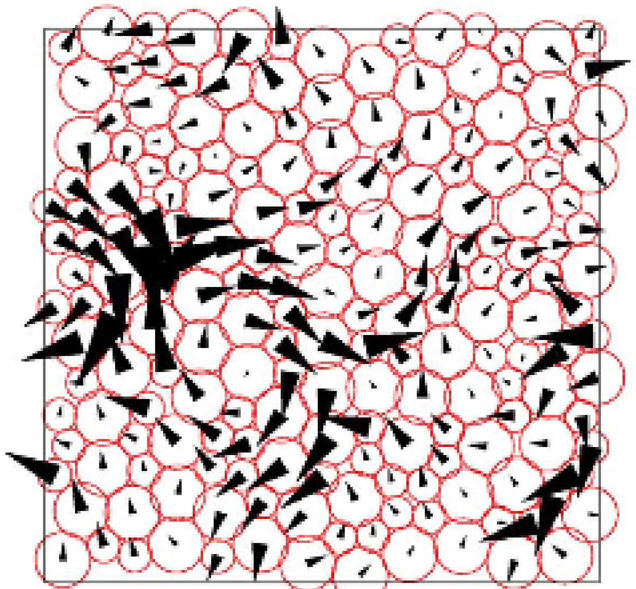


Johnson group, Caltech; ETH-METPHYS;
Oak Ridge National Lab; NPG Asia Materials (2011) 3, 82-90

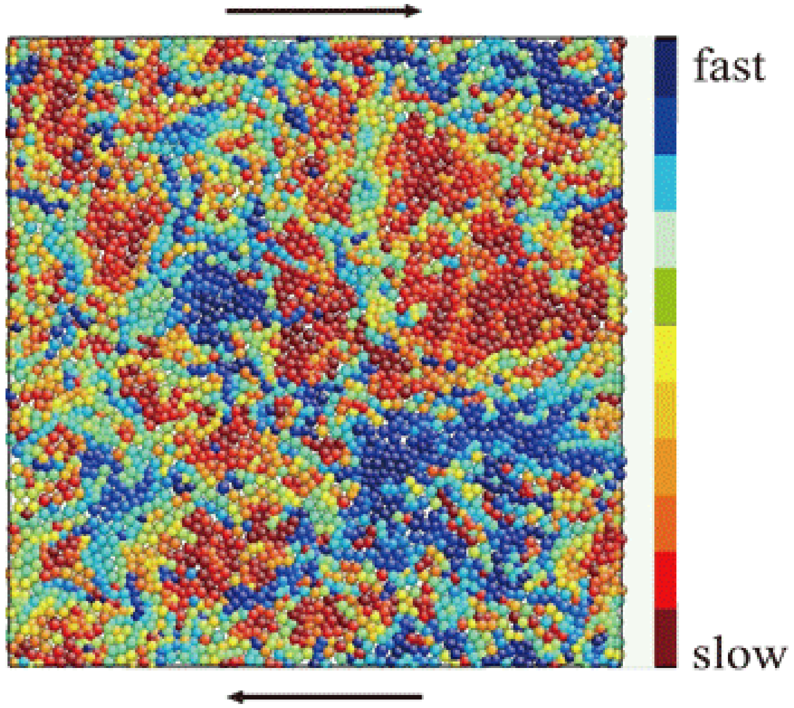
Amorphous materials

2d particle mixtures

University of Cambridge



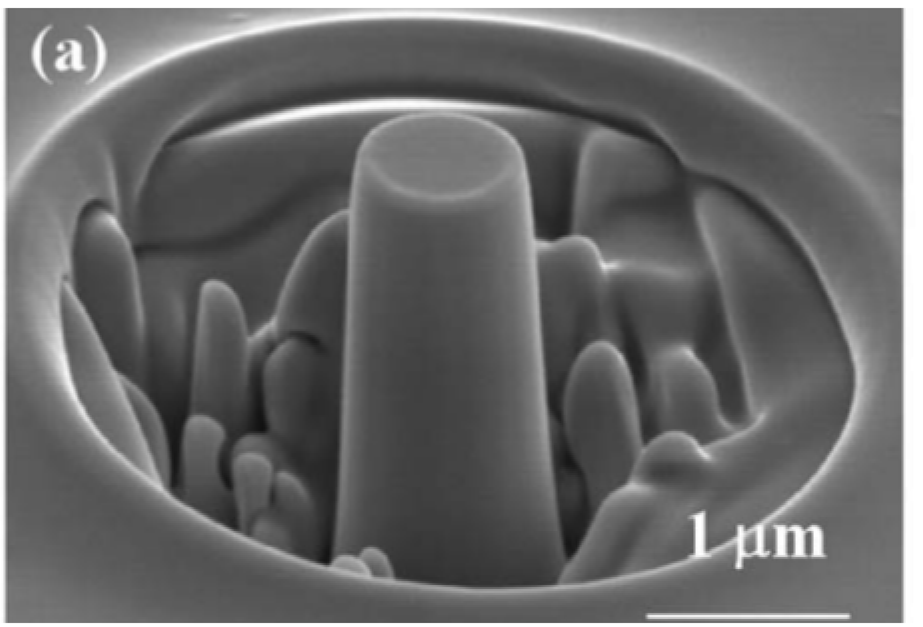
Maloney and Lemaitre,
Phys Rev E 74, 016118 (2006)

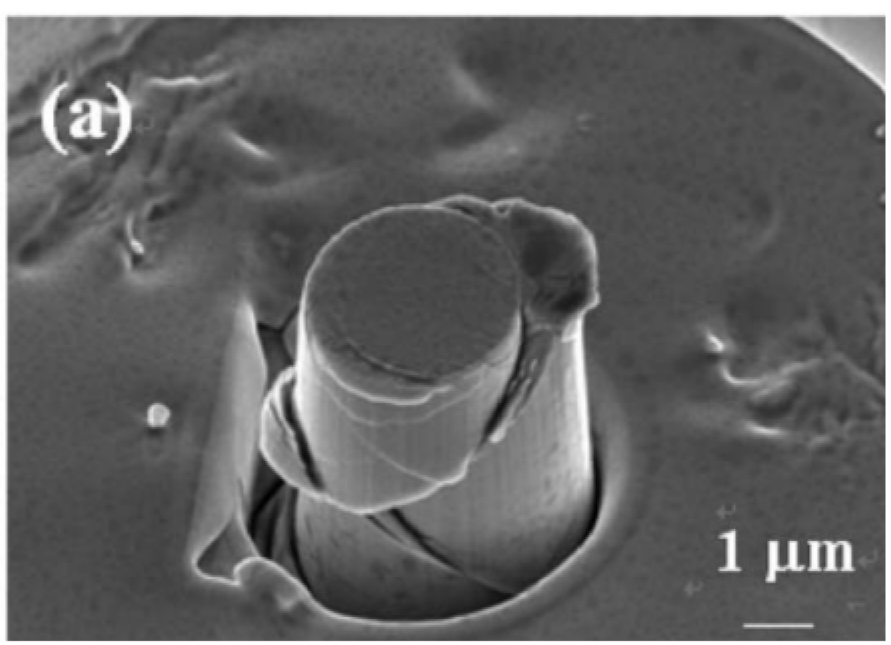


H. Hayakawa: JPSJ Online—News and Comments [December 10, 2008]

Deformation tests

Compression tests





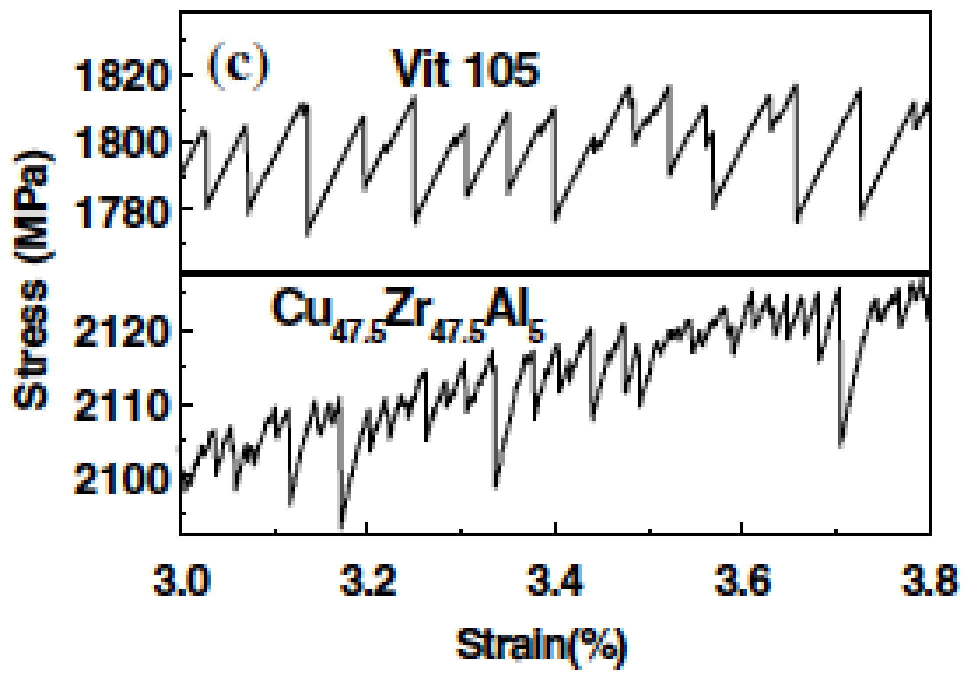
Lee et al, Appl. Phys. Lett. 91 161913 (2007)

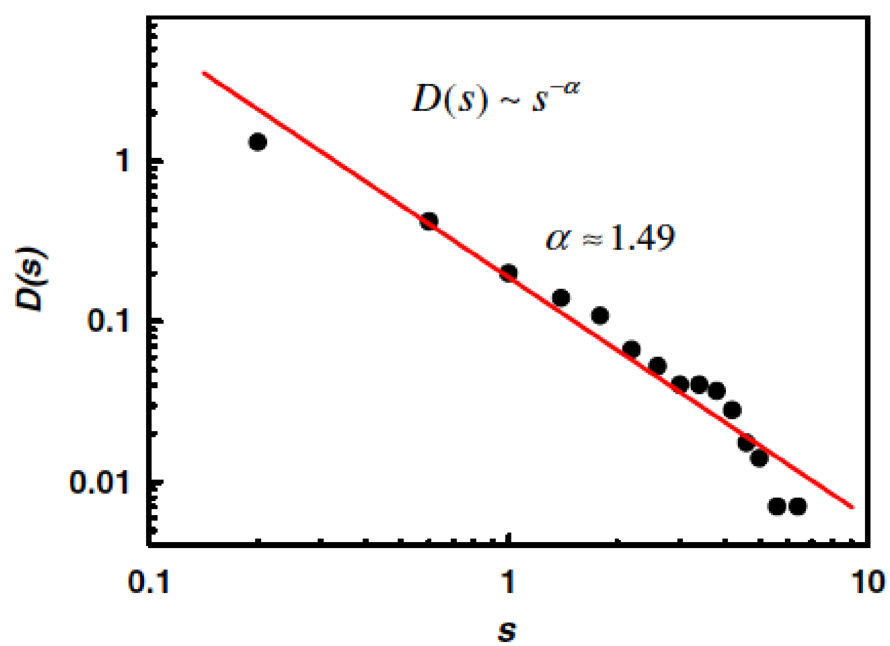
Shear in Couette cell

S. Schöllmann, S. Luding
http://www.icp.uni-stuttgart.de/movies/

Avalanches in amorphous plasticity

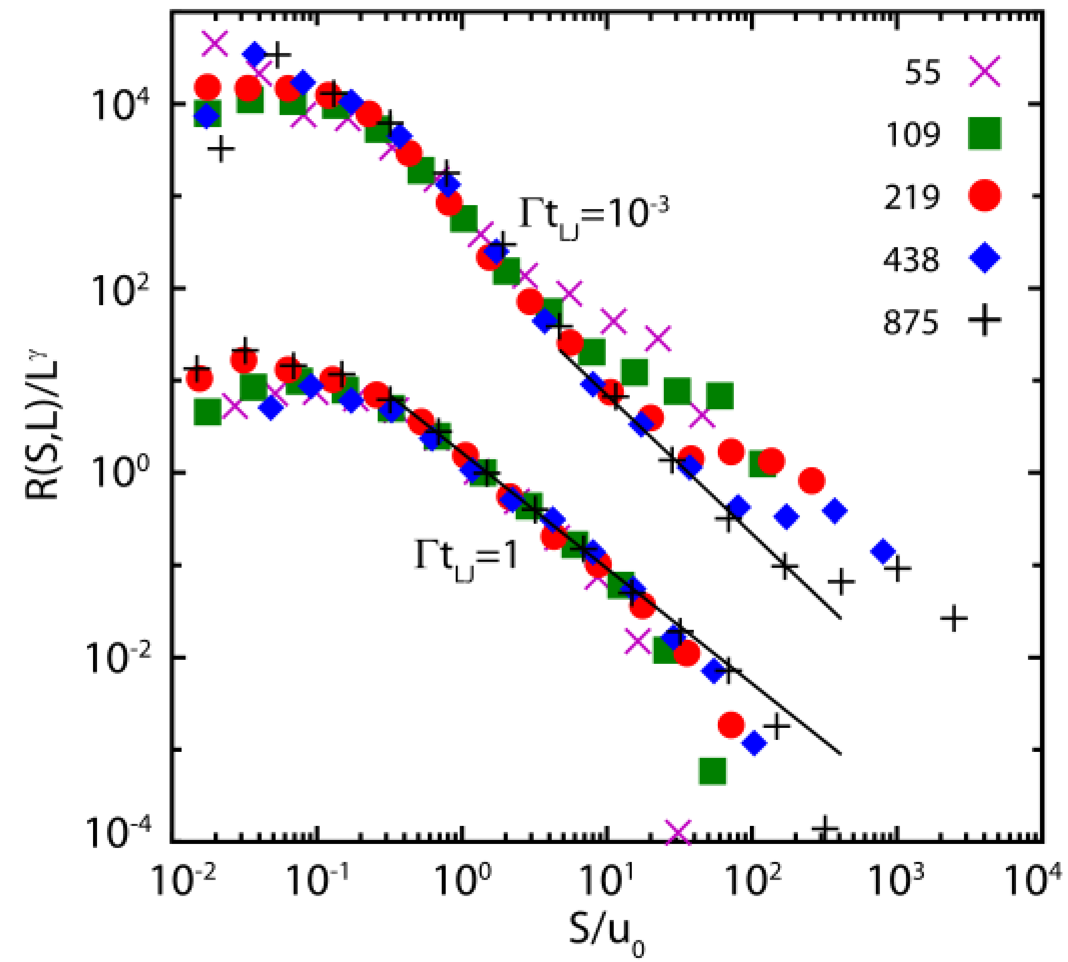
Bulk metallic glasses





Sun et al PRL 105, 035501 (2010)

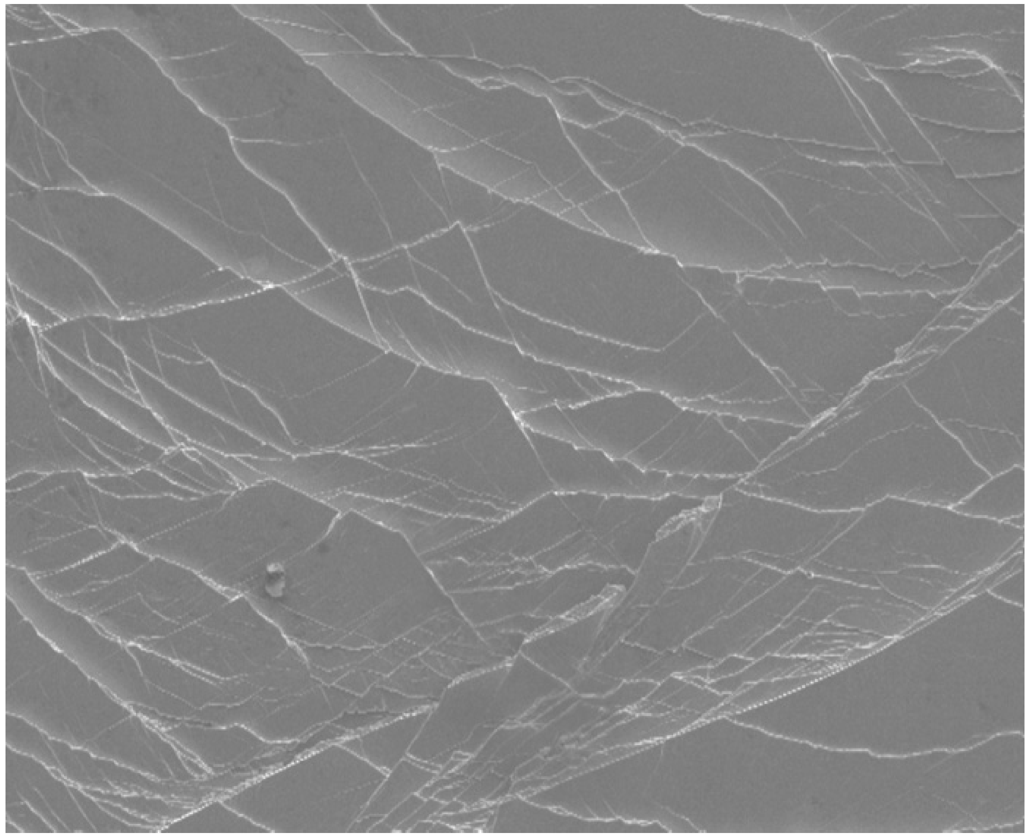
2d particle mixtures



Salerno et al PRL 109 105703 (2012)

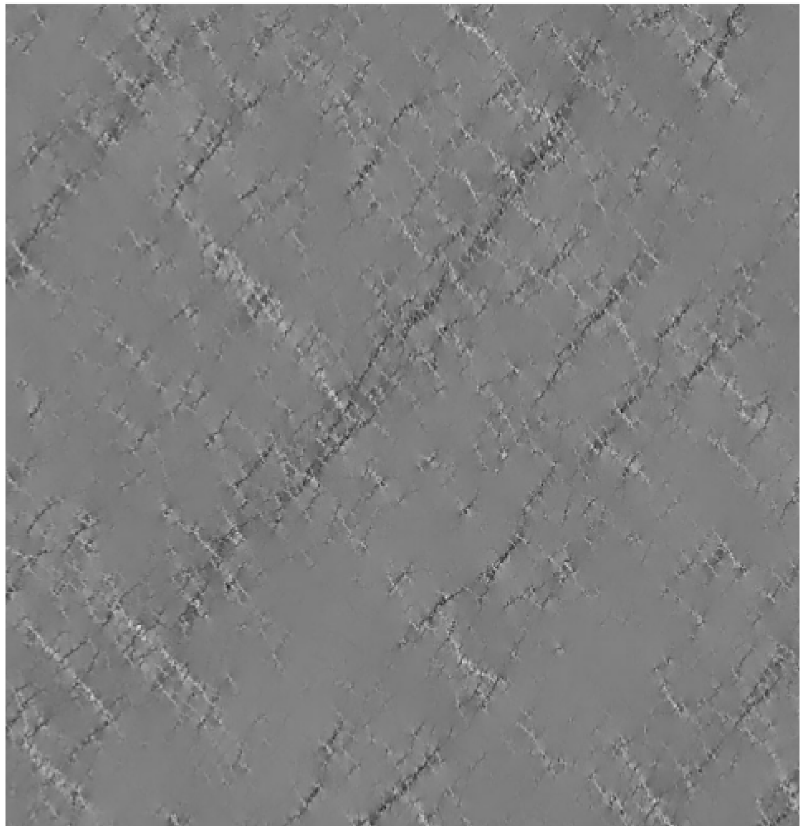
Strain localization

Bulk metallic glasses



Sun et al. Appl. Phys. Lett. 98, 201902 (2011)

2d particle mixtures



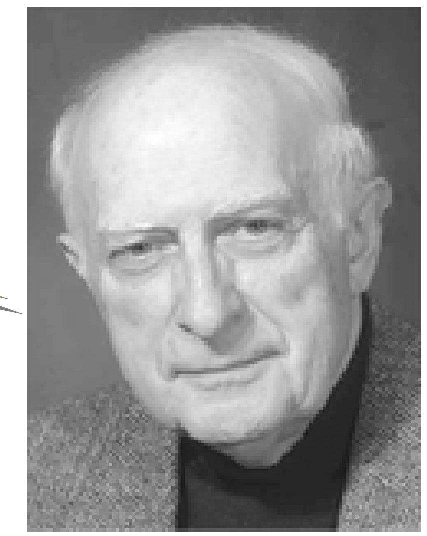
Maloney & Robbins,
PRL 102 225502 (2009)

A coarse-grained picture:

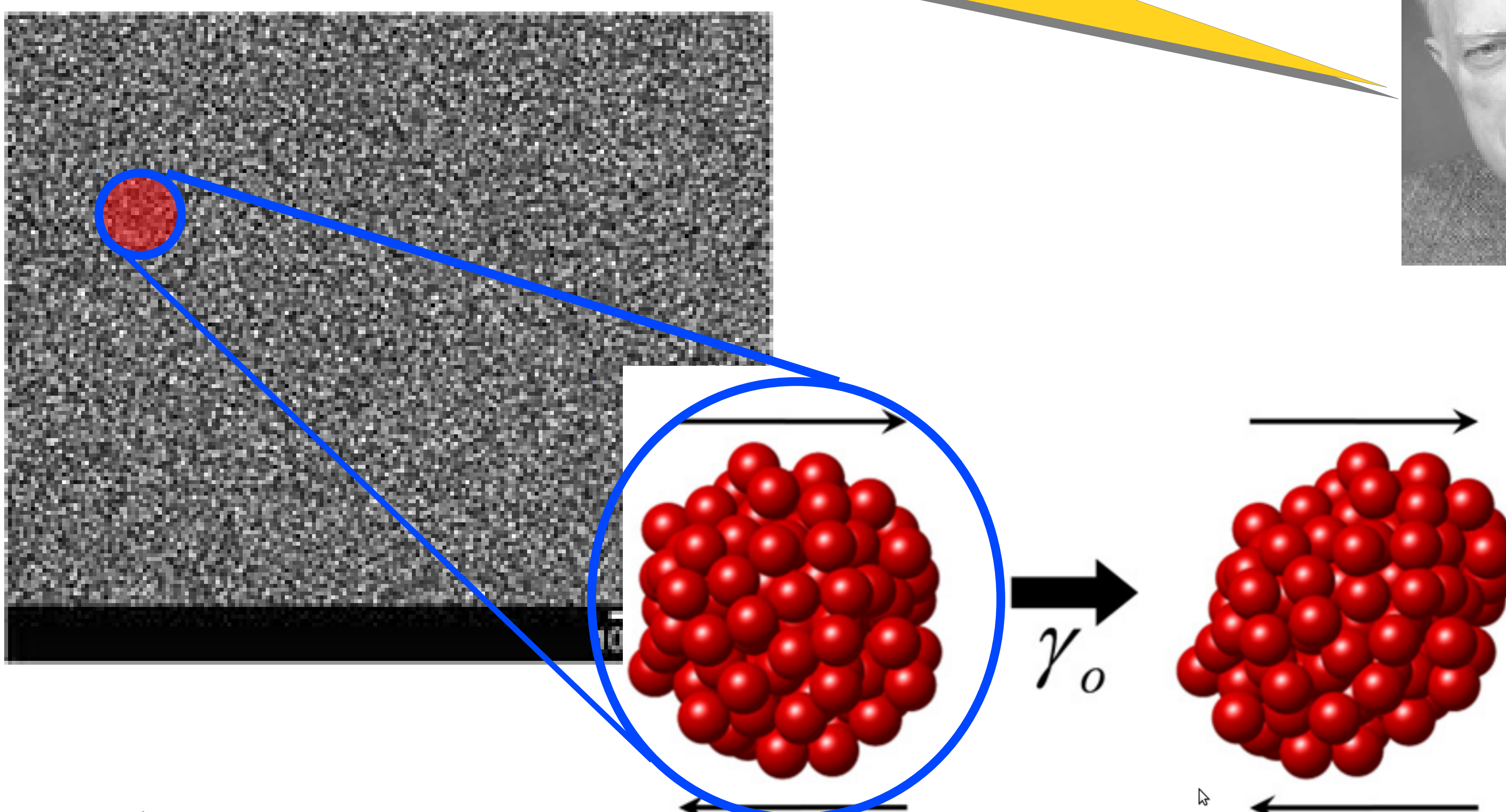
Lattice-based depinning model

How do amorphous materials deform?

Shear transformations



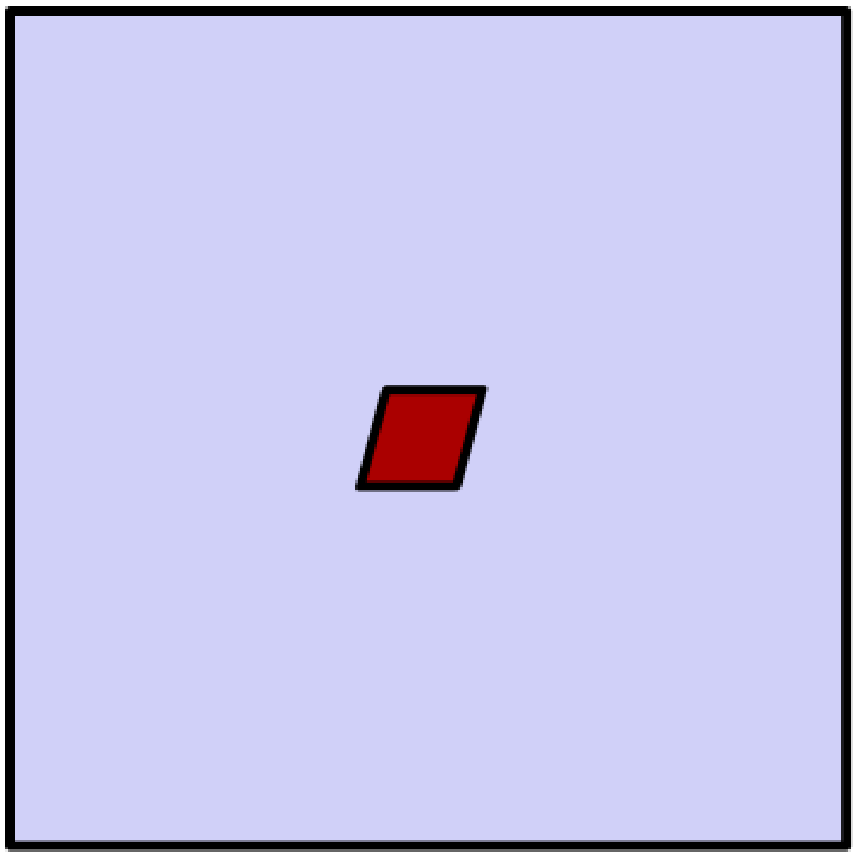
Argon



Particle rearrangement
→ relieves stress

Homer & Schuh, Modelling Simul. Mater. Sci. Eng. 18 (2010) 065009

Long-range interactions



Inclusion: plastic strain $\boldsymbol{\varepsilon}$
Matrix: stresses $\boldsymbol{\sigma}$

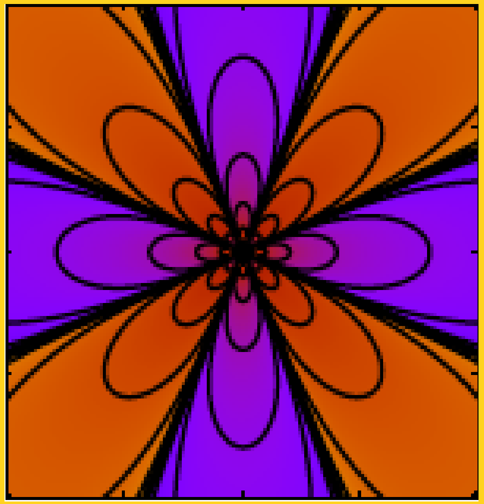
Linear elasticity → have Green's function:

$$\sigma(r) = \sum K(r - r')\epsilon(r')$$

Long-range interactions

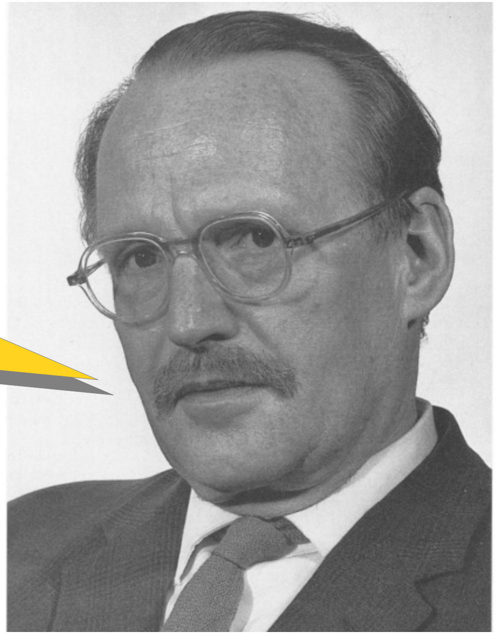
Pure shear → scalar Greens function

Pointlike Eshelby inclusion in 2d gives:



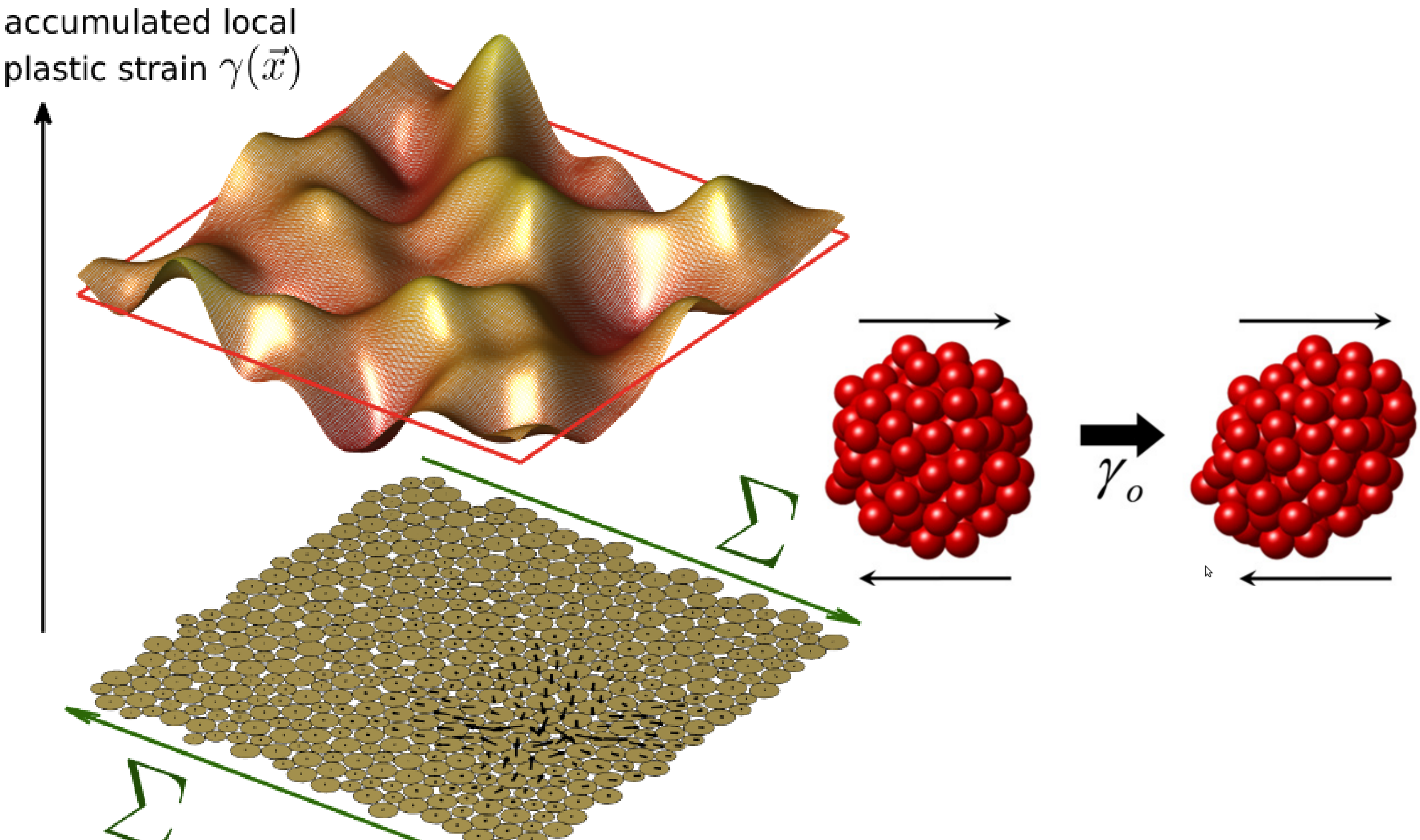
$$K(r) = \frac{\cos(4\theta)}{r^2}$$

- Long range
- Anisotropic



Eshelby

Collection of STZs → local strain map



Lin et al, arXiv:1403.6735

Modeling: depinning transition

3 ingredients in competition:

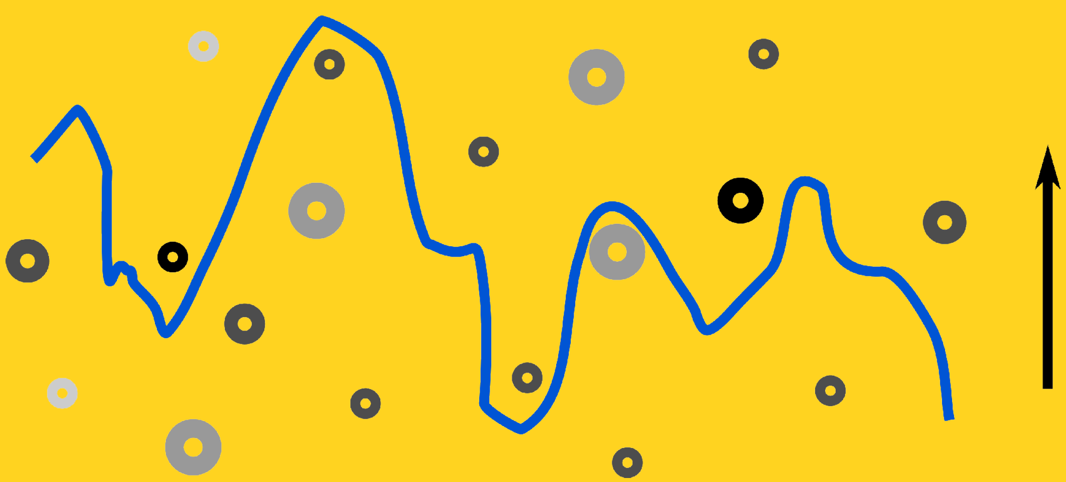
Interface depinning:

$$f_{\mathrm{tot}} = f + \nabla^2 u - \eta$$

External force

Interaction

Pinning potential



Amorphous plasticity:

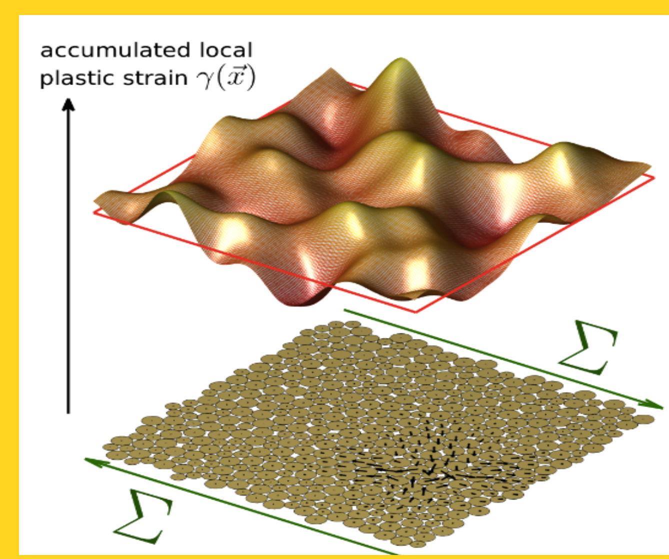
External stress σ_{ext}

- Internal stress

$$\sigma_{int}(r) = \int d^2r'(\gamma(r') - \gamma(r))K(r' - r)$$

- Random local yield thresholds

$\sigma_c(r)$



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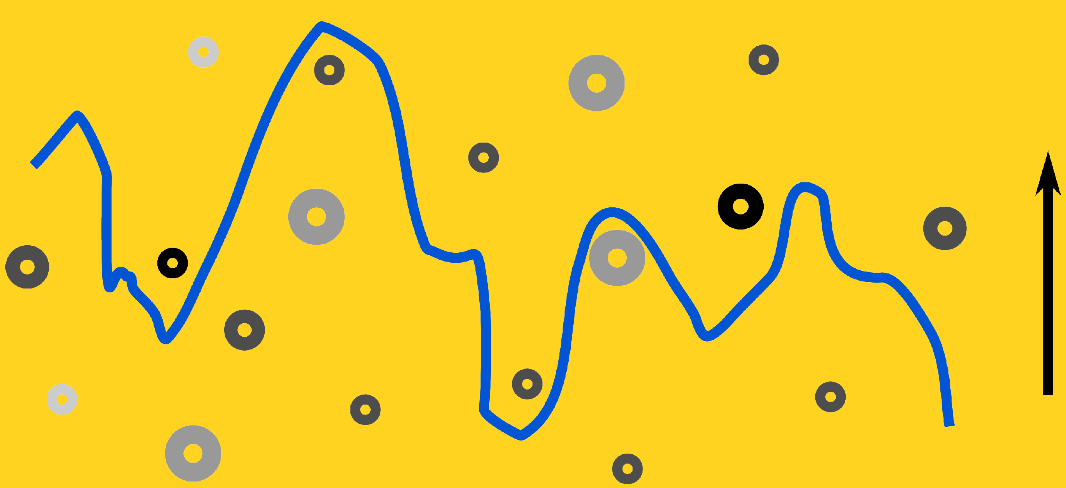
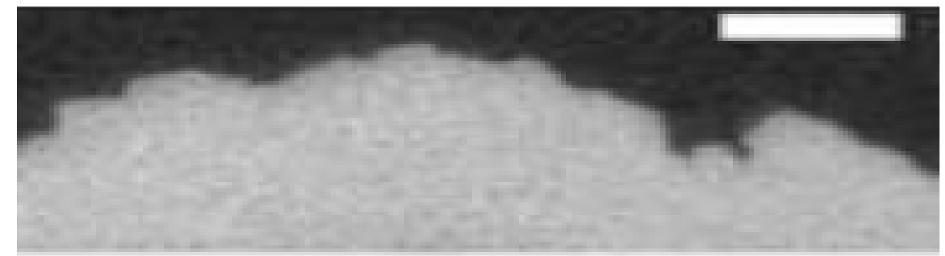
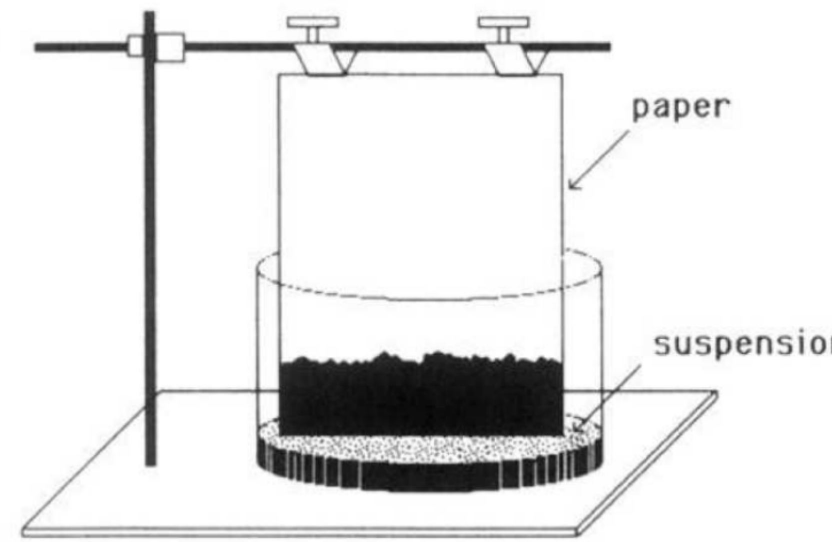
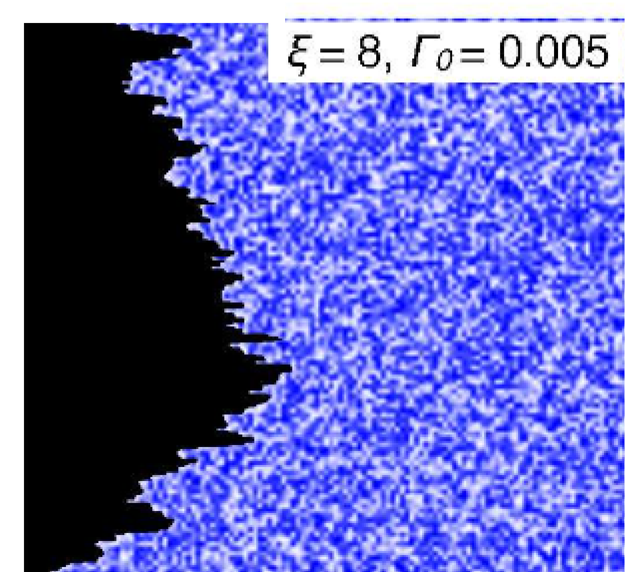




Metaxas et al, PRL 99, 217208



Buldyrev et al, PRA 45, R8313



Laurson & Zapperi, J. Stat. Mech. (2010) P11014

Modeling: depinning transition

3 ingredients in competition:

Interface depinning:

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External force

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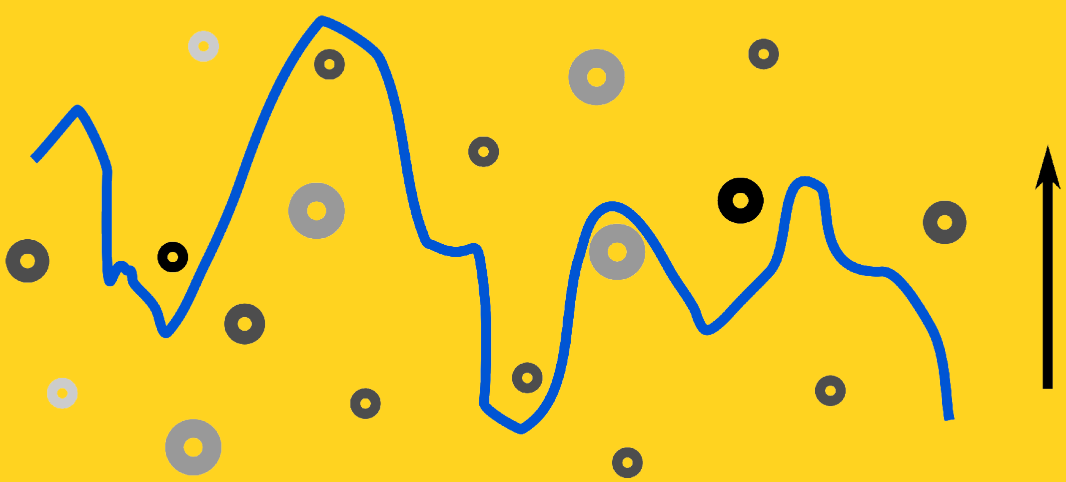
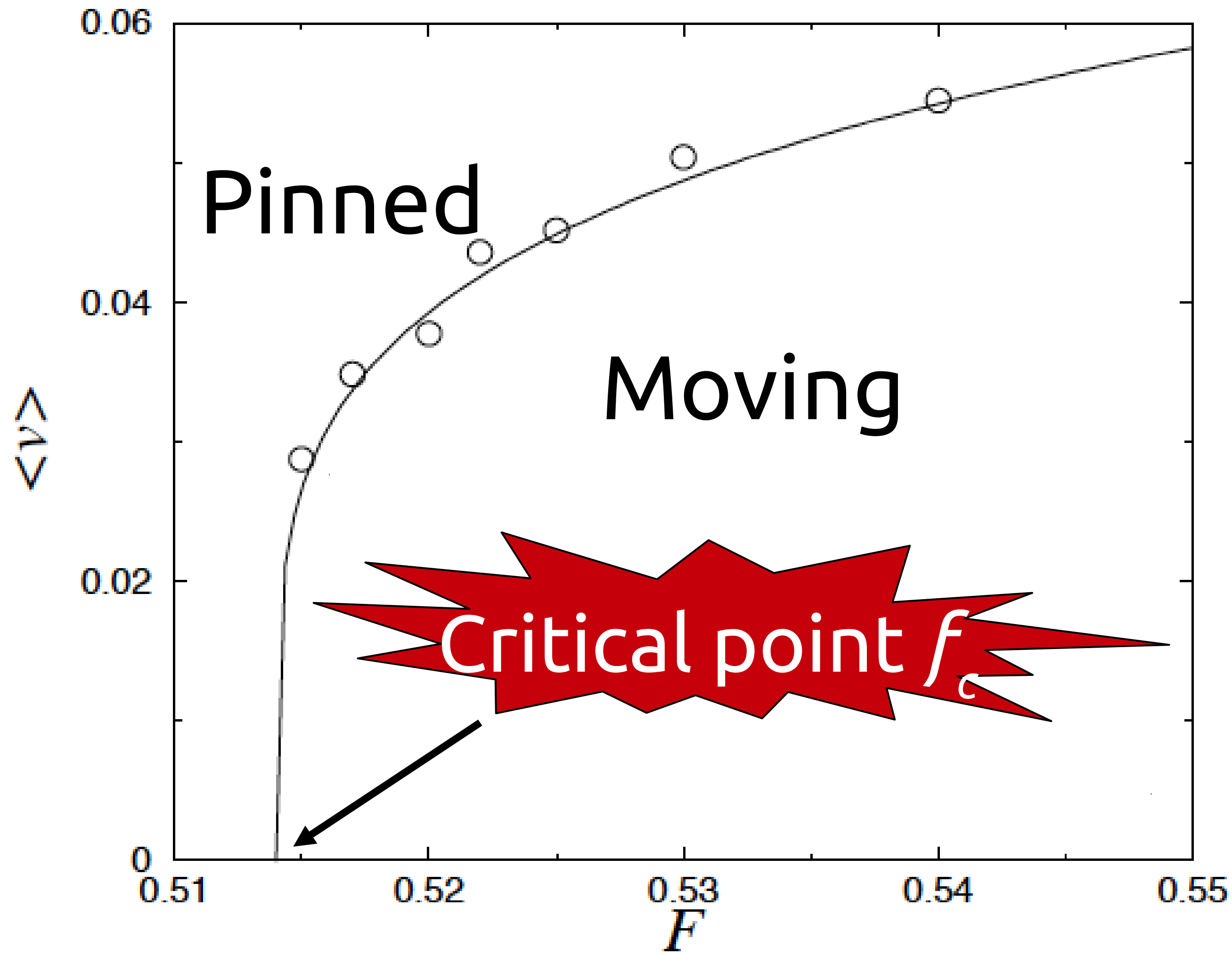


Phase transition:



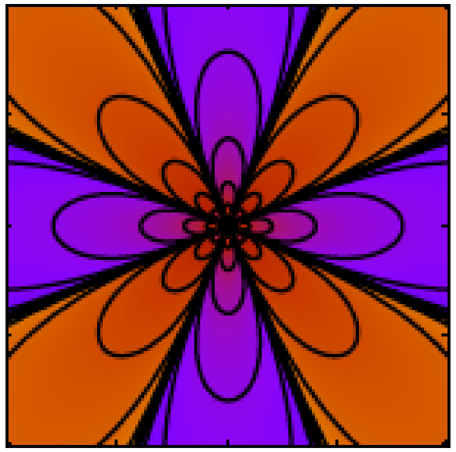
At criticality: **power-law distributed avalanches**, diverging size cutoff

2 questions:

What is universality class?

What about localization?

Depinning transition universality class



Internal stresses are long range:

$$K(r) = \frac{\cos(4\theta)}{r^2} \qquad \tilde{K}(q) = \frac{q_x^2 q_y^2}{q^4}$$

Functional Renormalization Group:

If kernel in Fourier space is $\tilde{K}(q) = q^{\alpha}$

Then upper critical dimension is $d_c = 2\alpha$

For $d > d_c$ mean-field theory holds

$$P(s) = s^{-3/2} \exp(-s/s_0)$$

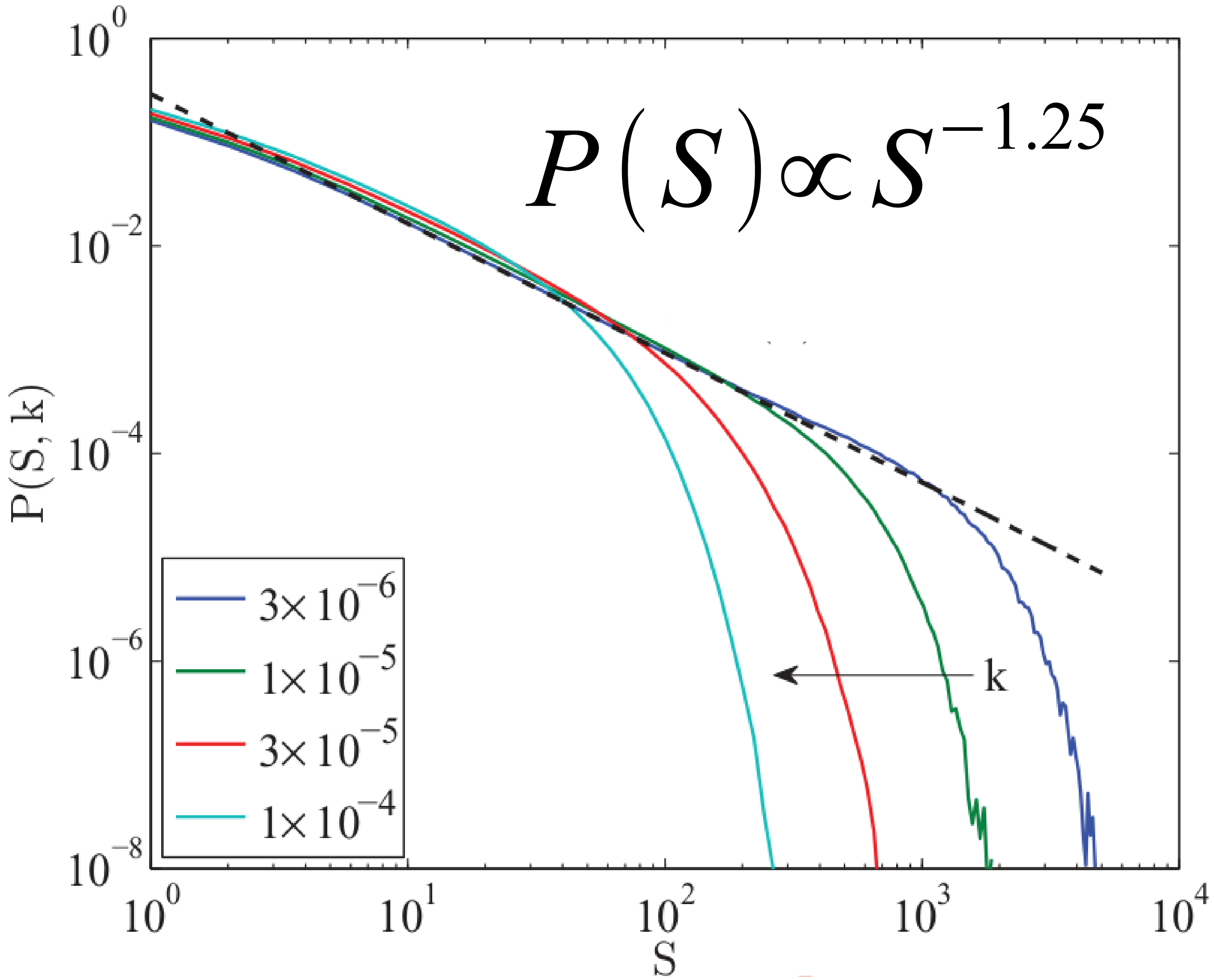
$$s_0 \sim (f_c - f)^{-2}$$

?

Depinning transition universality class

But! FRG assumes convex interaction kernel (“no passing” rule)

Simulations…?



Talamali, Petäjä, Vandembroucq and Roux, PRE 84 016115 (2011)

2d simulations:

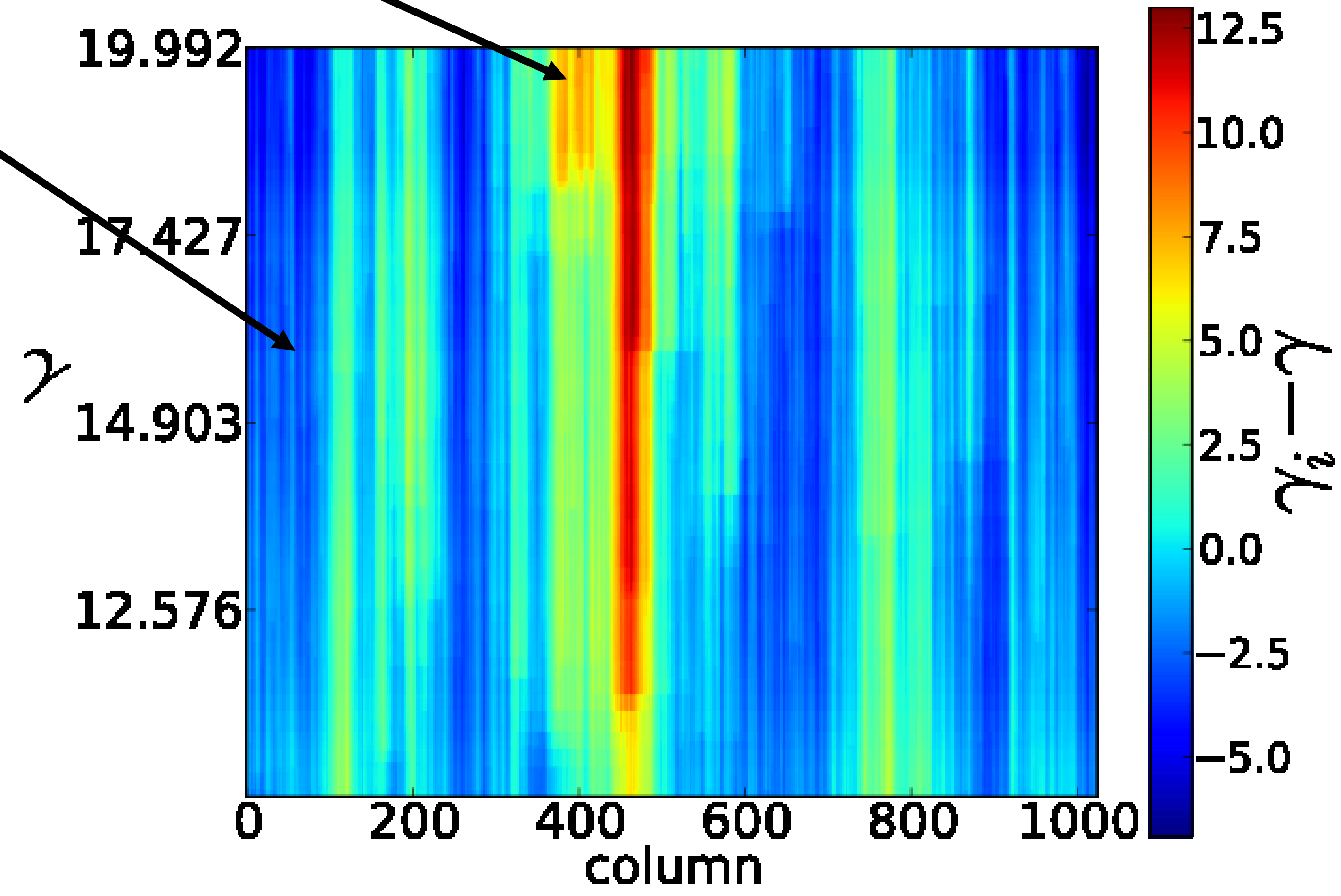
Non-universal crossover from mean field

Large scale simulations

Adiabatic driving
Discrete time, instantaneous stress redistribution
Pure shear
1024x1024 lattice, periodic BCs

Strain localization

Features can emerge and disappear



Periodic boundary conditions

Method 1: Fourier

-Fourier transform in infinite system

-Discretize in Fourier space

-Transform back to real space

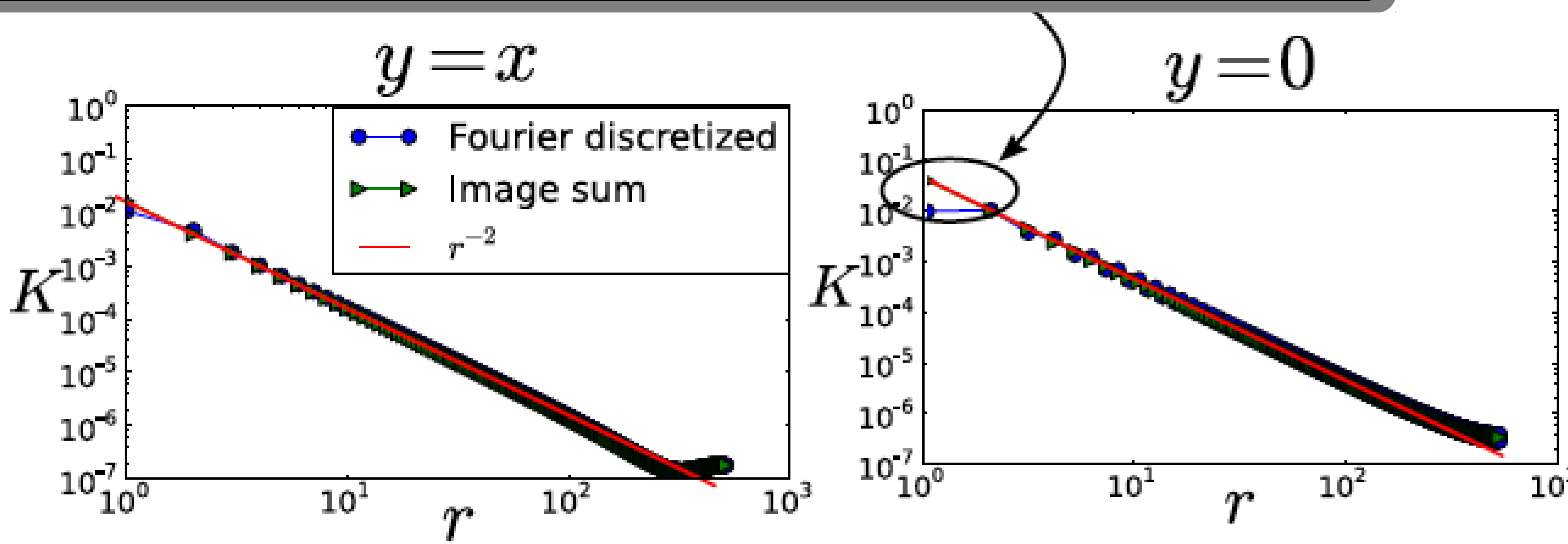
(Talamali et al)

Method 2: Image sum

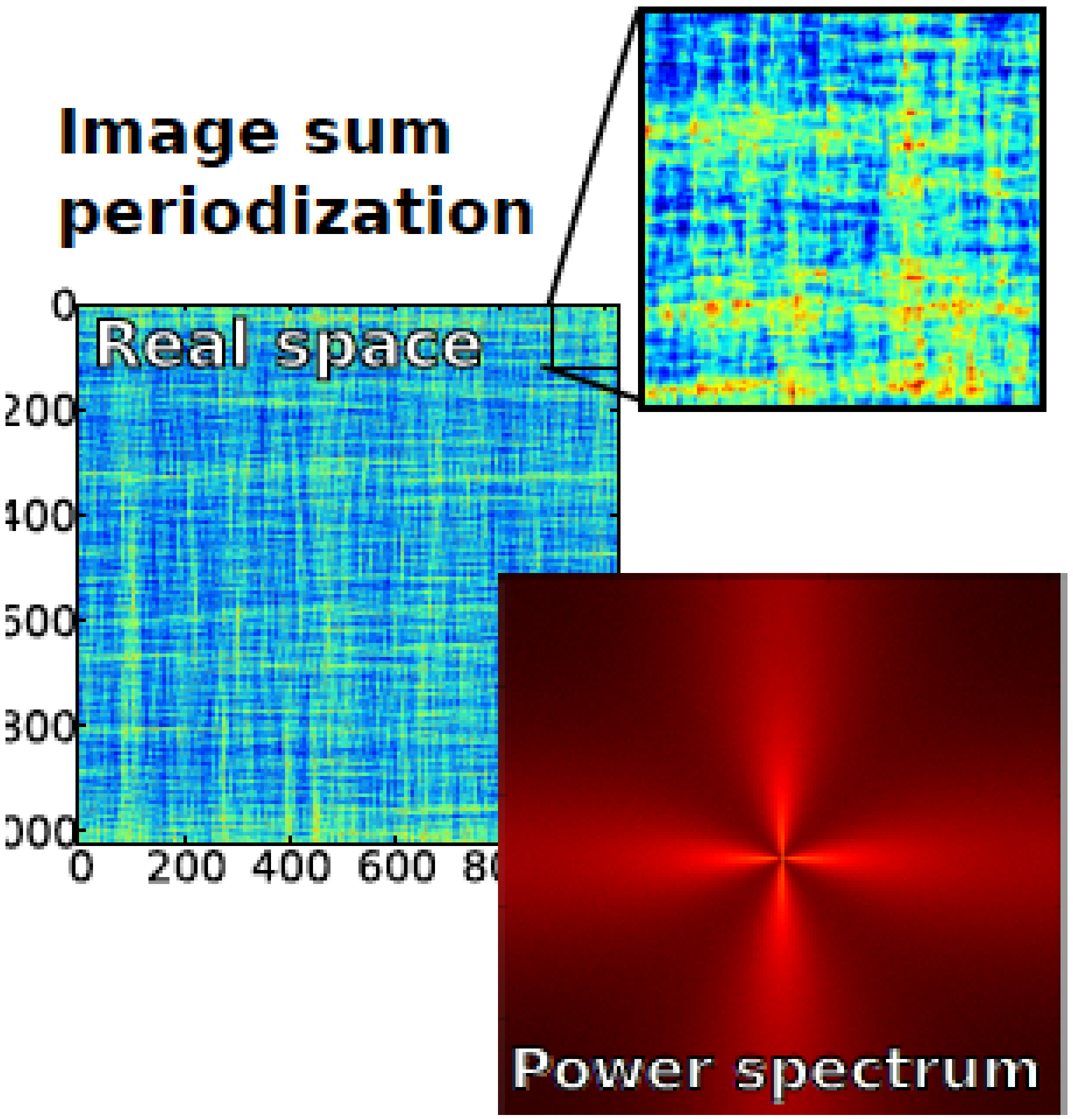
-Sum over infinite images in *y*

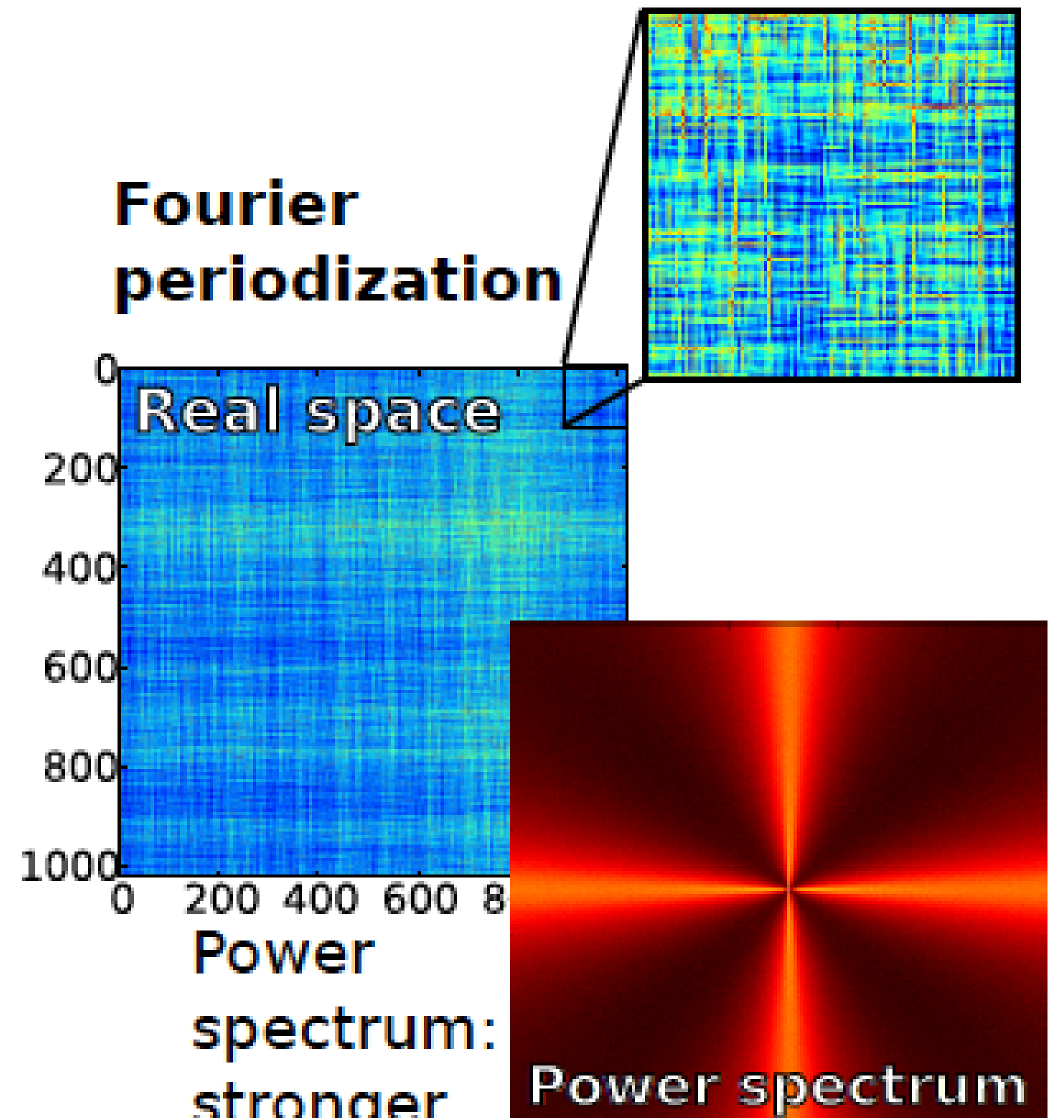
-Sum over fast-decaying terms in *x*

Same long range behavior but **different at short-range**



Short range interactions: change localization

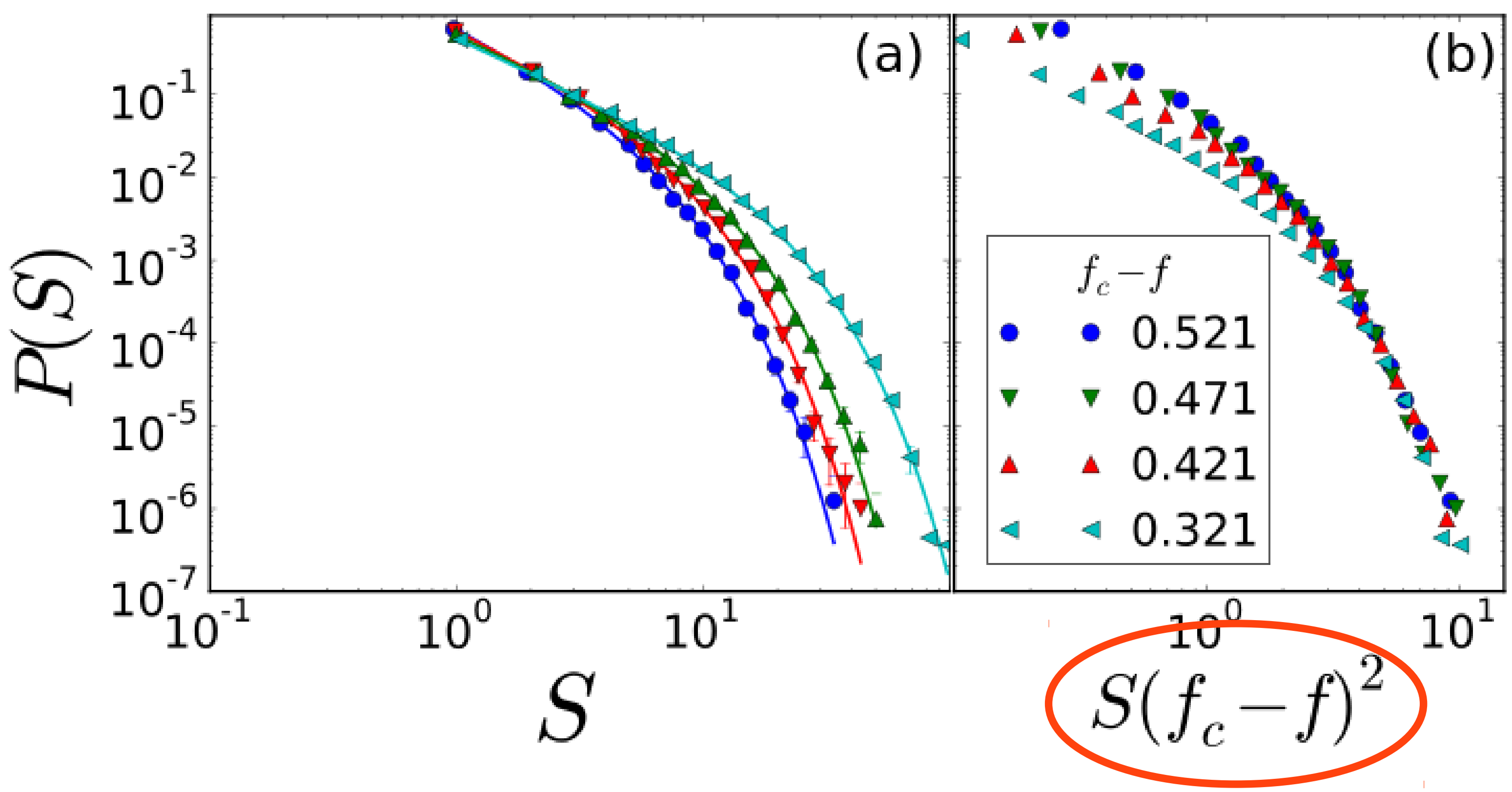




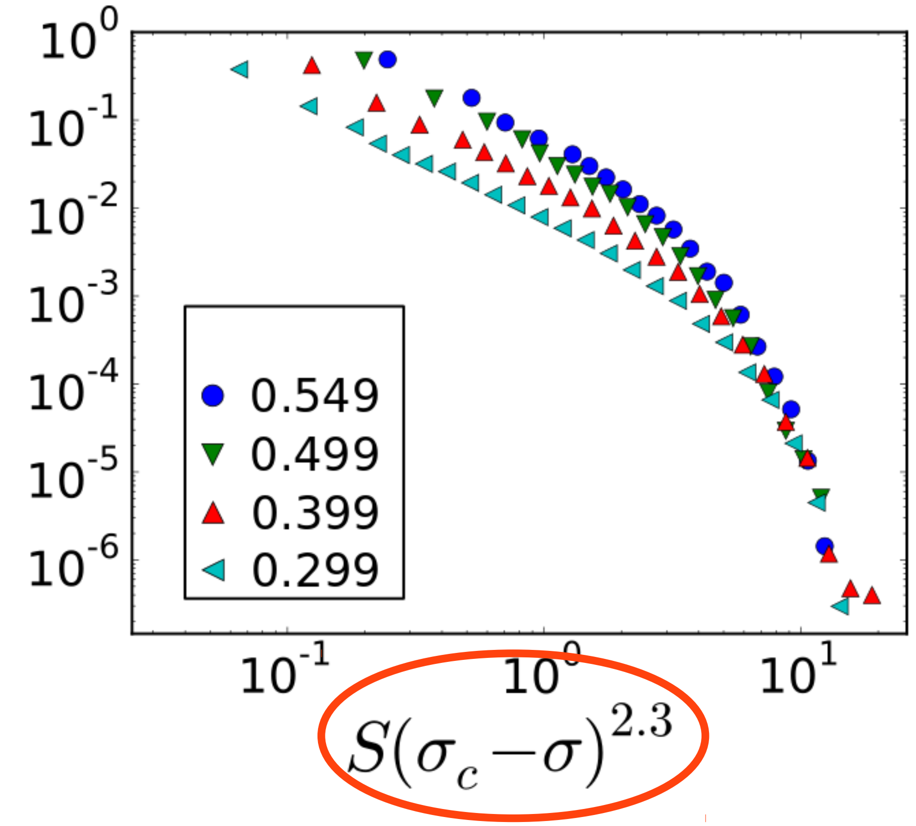
Power spectrum: stronger angular depencene

Short range interactions: non-universal behaviour at small stresses

Image sum kernel



Fourier kernel

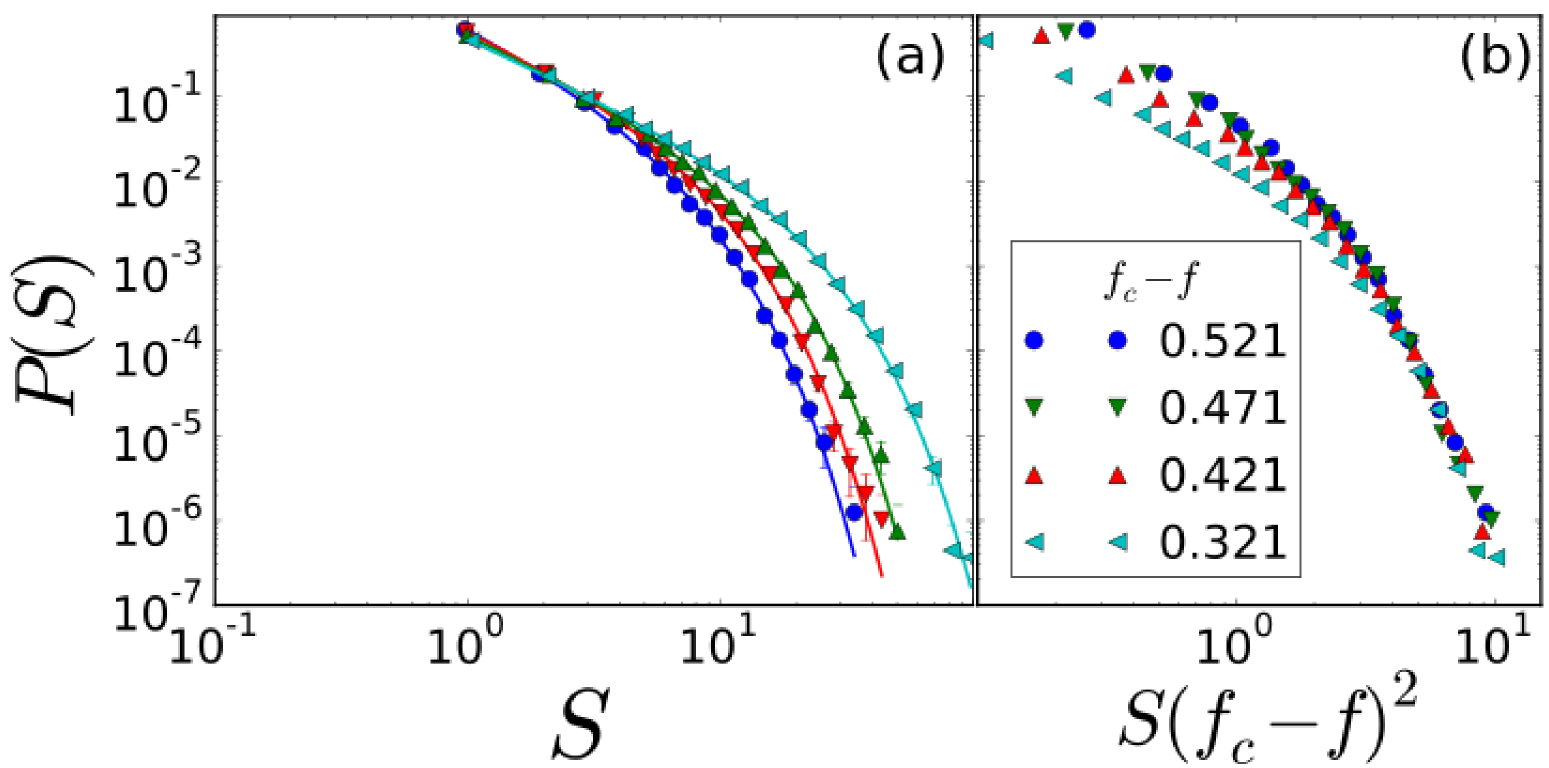


Mean field

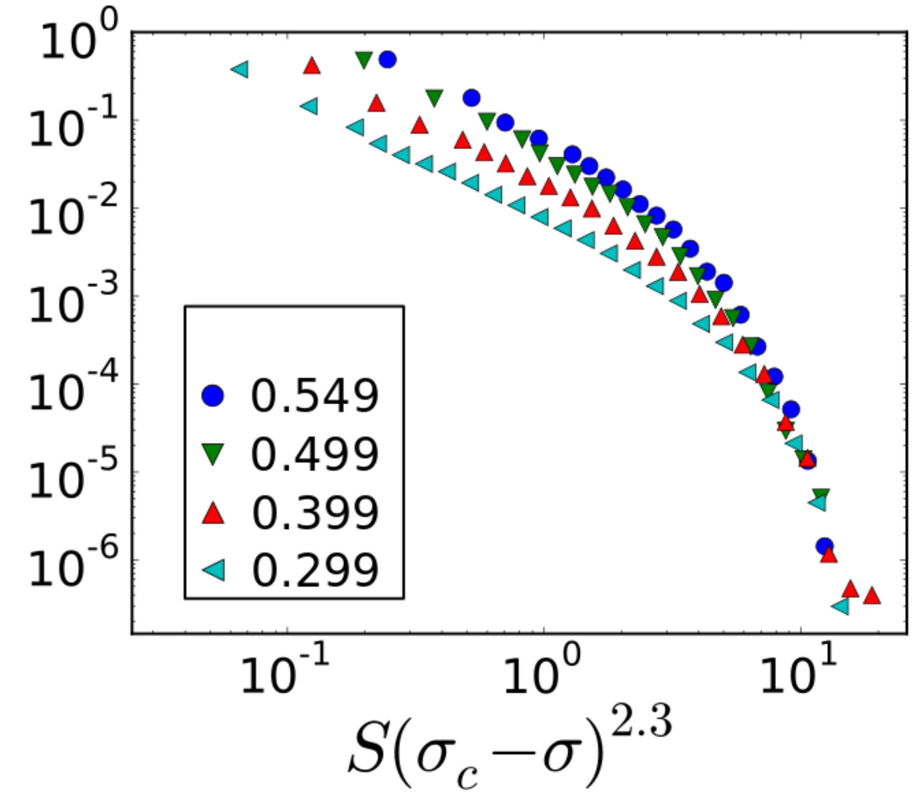
Not mean field

Short range interactions: non-universal behaviour at small stresses

Image sum kernel



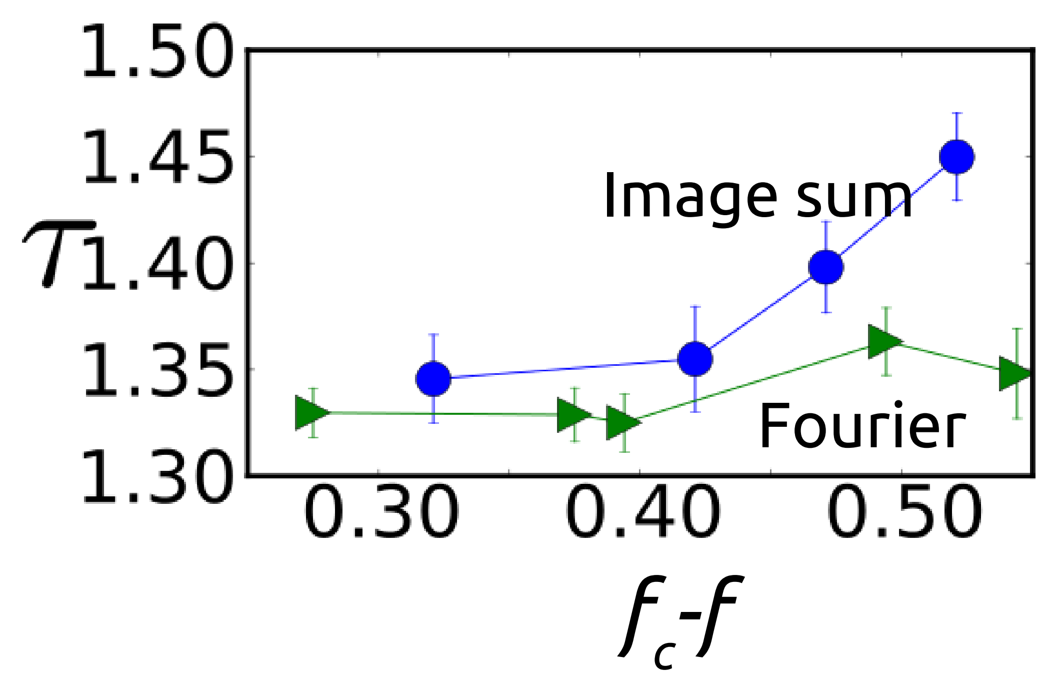
Fourier kernel



Fit with

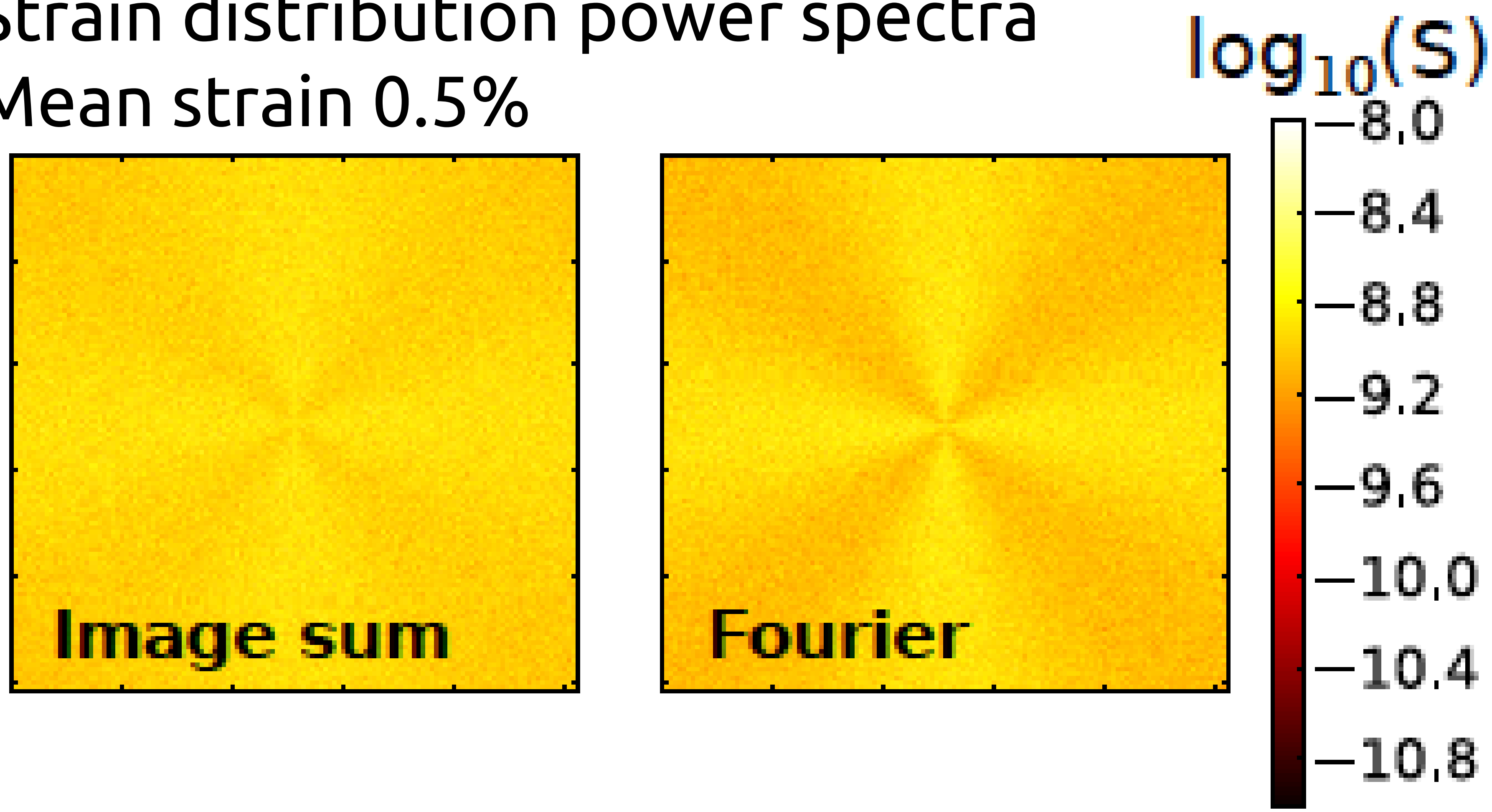
$$P(S) = c_1 S^{-\tau} \exp\left(c_2(-BS^{-\delta} + C\sqrt{S})\right)$$

Le Doussal & Wiese, PRE 85 061102 (2012)



Localization depends on short-range interactions

Strain distribution power spectra
Mean strain 0.5%

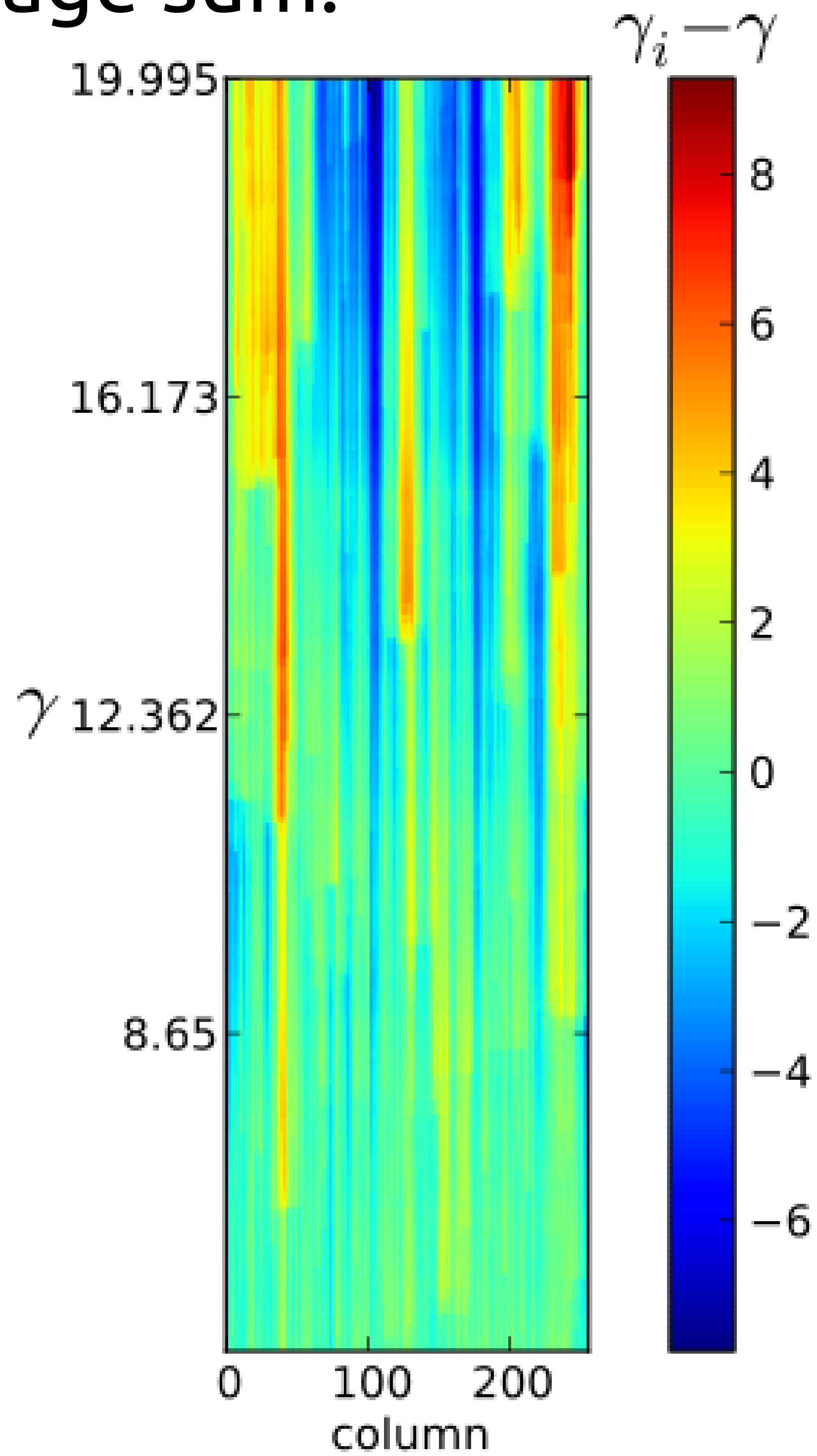


Fourier kernel shows stronger localization.

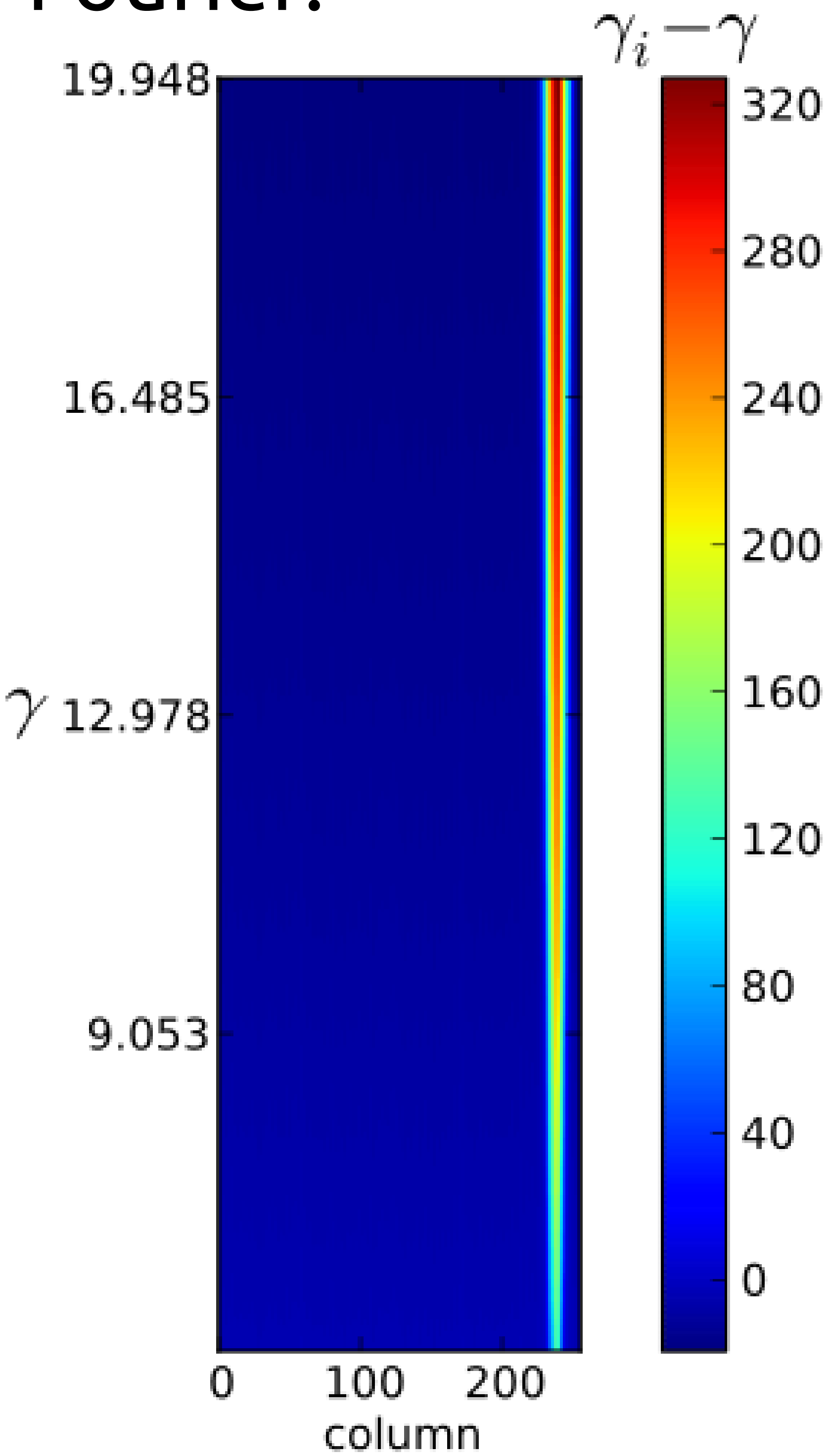
→ Origin of nonuniversal crossover?

Localization depends on short-range

Image sum:



Fourier:

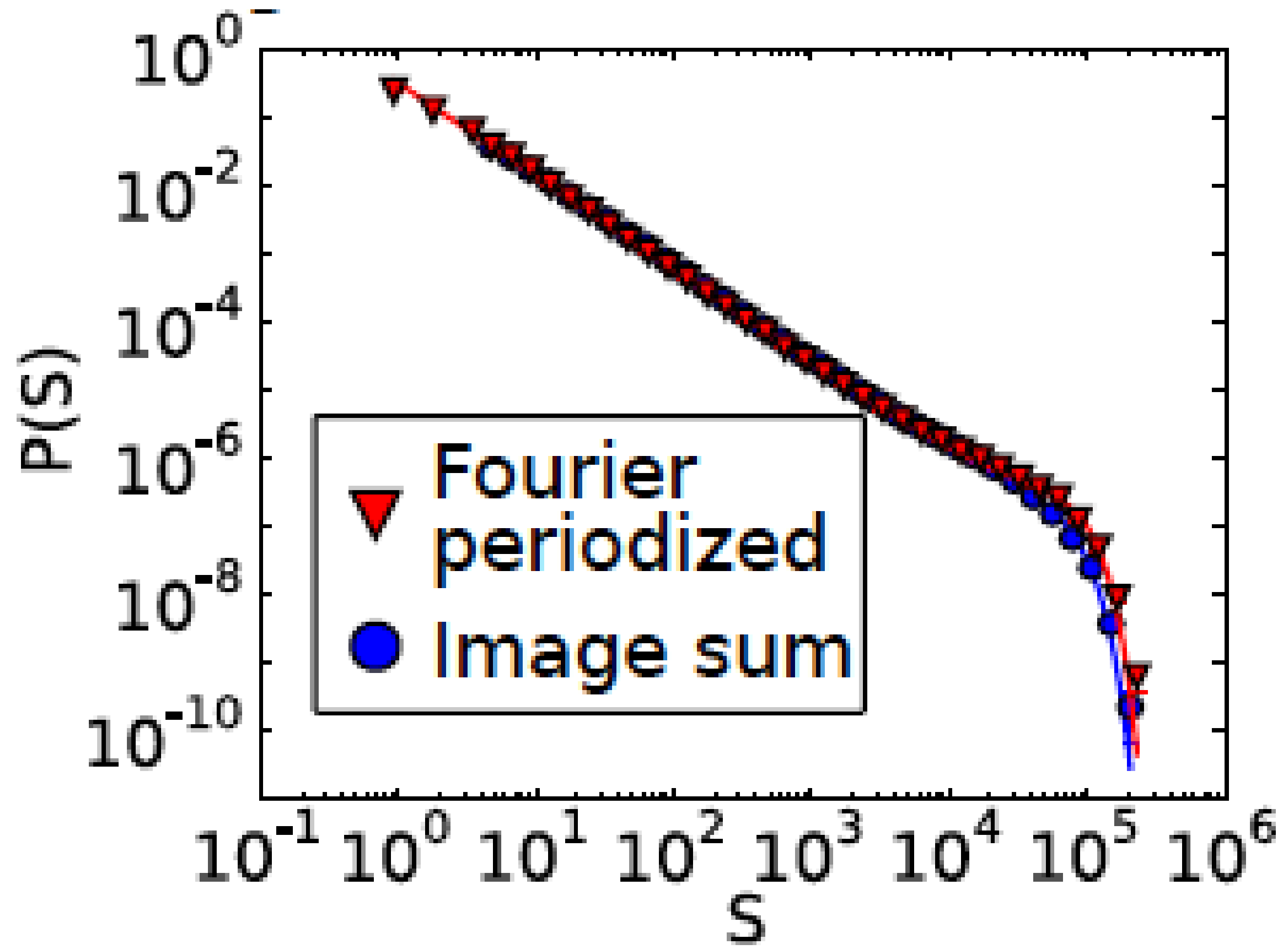


At criticality: universal, not mean field

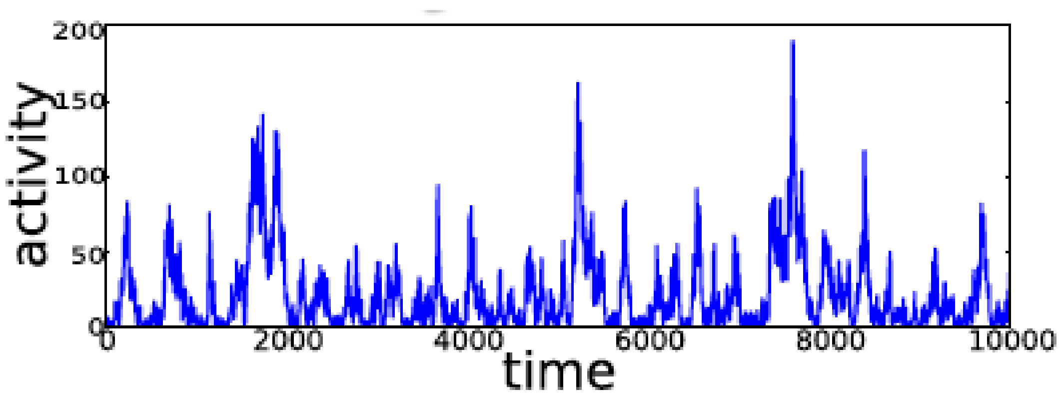
$$p(s) = as^{-\tau}\exp(bs - cs^2)$$

Not mean field:

$\tau = 1.35$



Power spectrum



Bursty activity
Power law distribution
Universal

$$PS(\omega) \sim \omega^{-1/\sigma\nu z}$$

$$\langle S(T) \rangle \sim T^{1/\sigma\nu z}$$

$$1/\sigma\nu z(MF) = 2$$

Fourier periodized kernel:

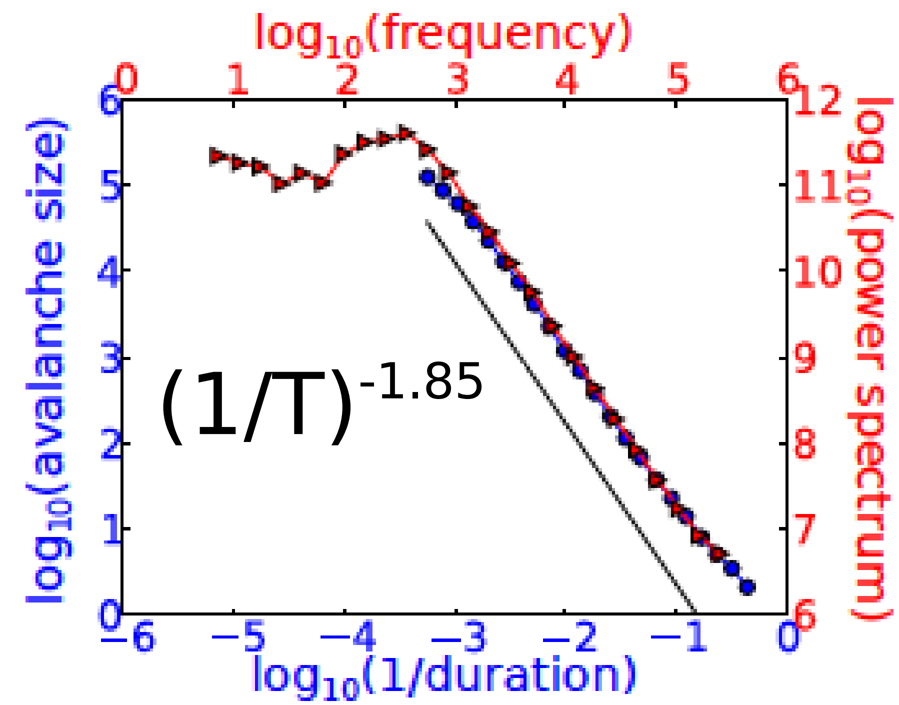
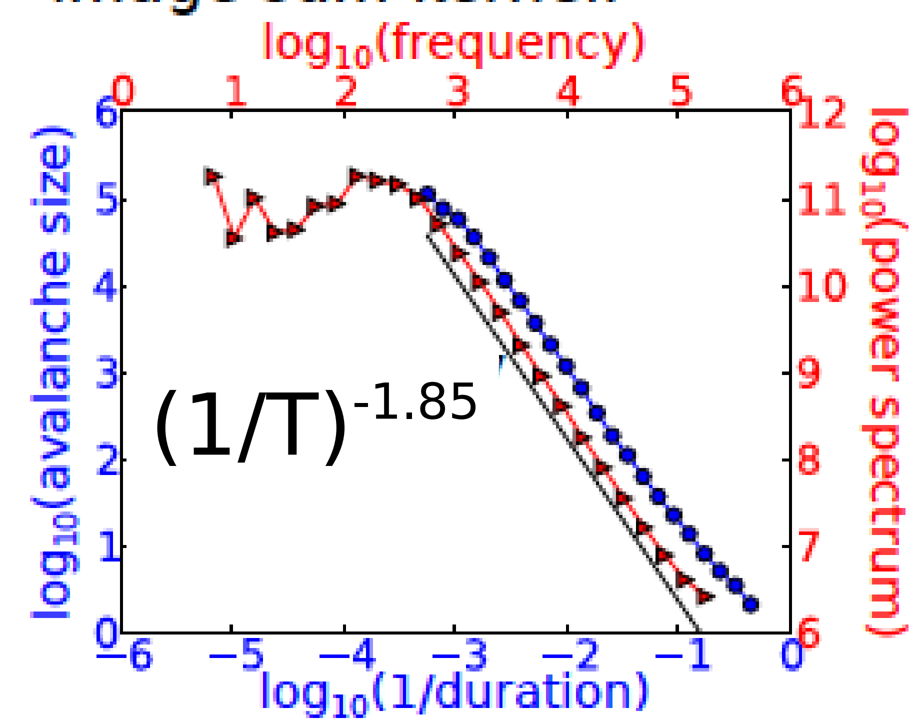


Image sum kernel:



Universality class: not mean field!

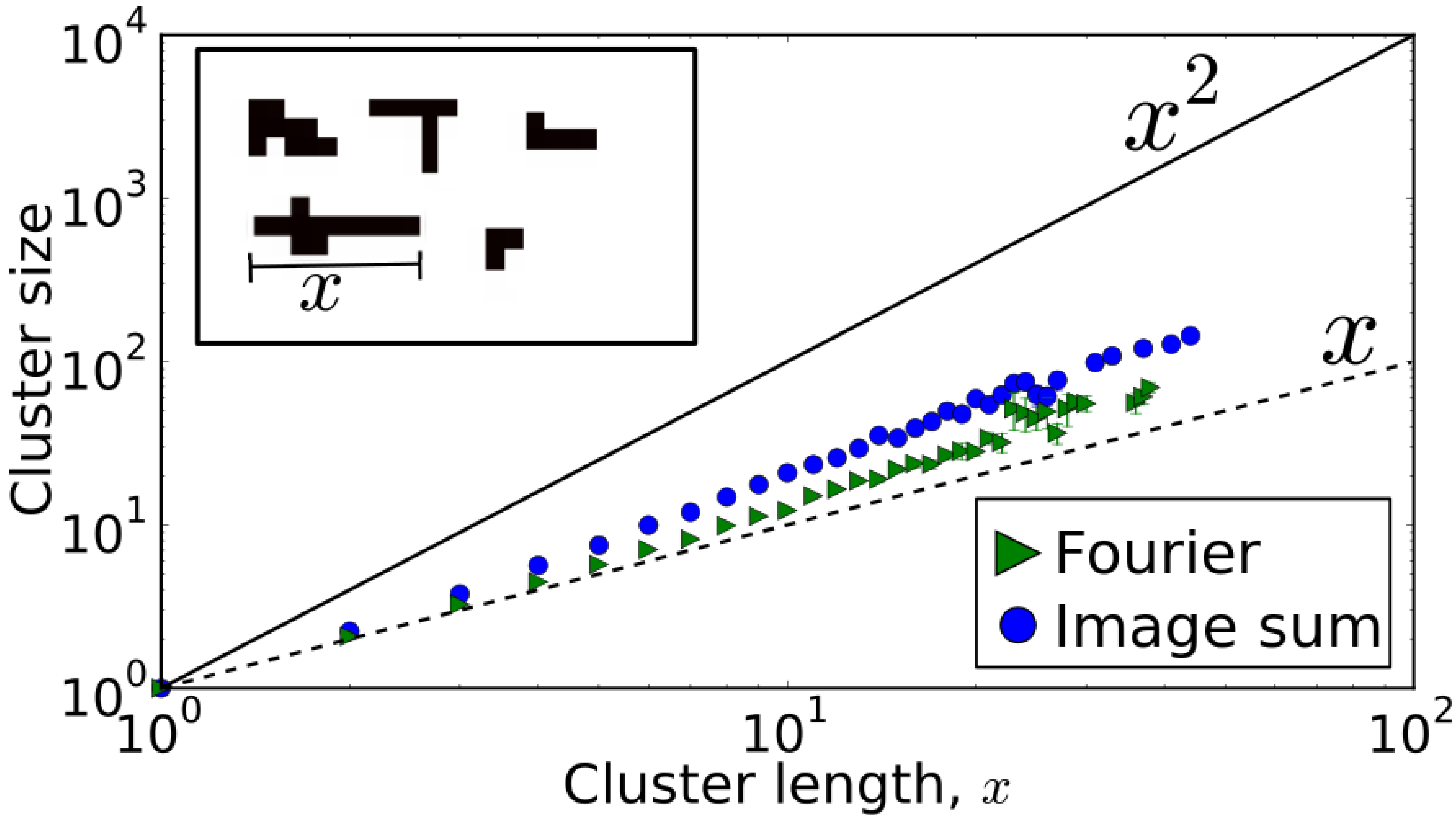
		2d	MF
Size distribution	τ	1.342±0.004	3/2
Size cutoff	$1/\sigma$	2.3±0.05	2
Duration distribution	α	1.5±0.09	2
Power spectrum	$1/\sigma\nu z$	1.85±0.05	2

Dimensional reduction?

		2d	MF	1d-LR
Size distribution	τ	1.342±0.004	3/2	1.25±0.05
Size cutoff	$1/\sigma$	2.3±0.05	2	2.1±0.08
Duration distribution	α	1.5±0.09	2	~1.43
Power spectrum	$1/\sigma\nu z$	1.85±0.05	2	~1.7

Dimensional reduction?

Avalanches are made of “clusters” with d>1

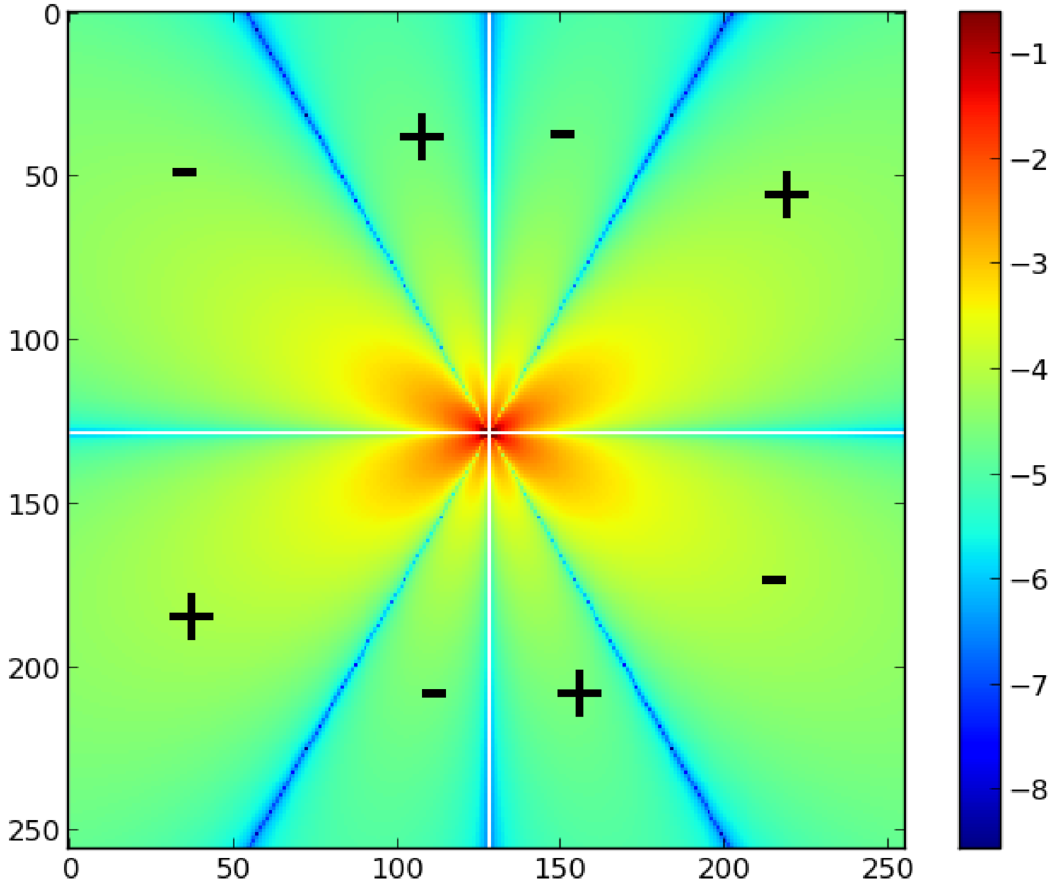


Tensorial models:

Beyond the scalar approximation

Why tensorial models?

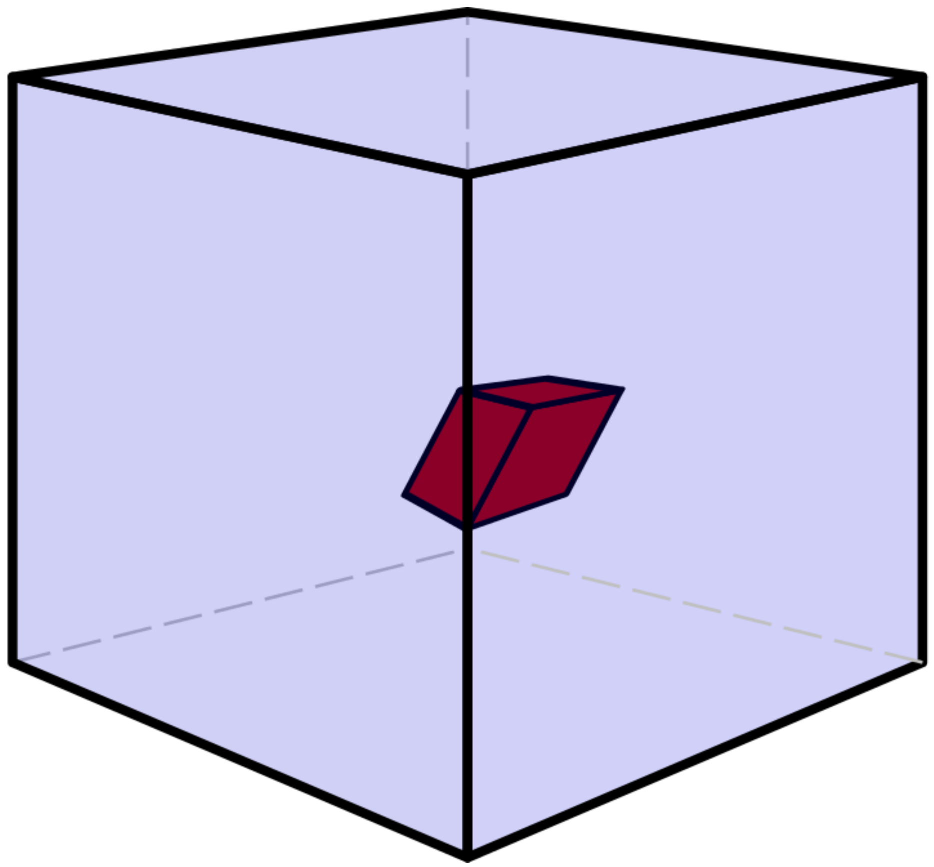
stress $\boldsymbol{\sigma}_{xx}$ due to plastic strain $\boldsymbol{\varepsilon}_{xy}$



Local stresses **not** pure shear!
→ plastic yield isn't pure shear

Generalized loading conditions?

3d simulations



Matrix: stresses $\boldsymbol{\sigma}$
Inclusion: plastic strain $\boldsymbol{\varepsilon}$

Tensorial Greens function

$$\sigma_{ij}(r) = \sum K_{ijkl}(r - r')\,\epsilon_{kl}(r')$$

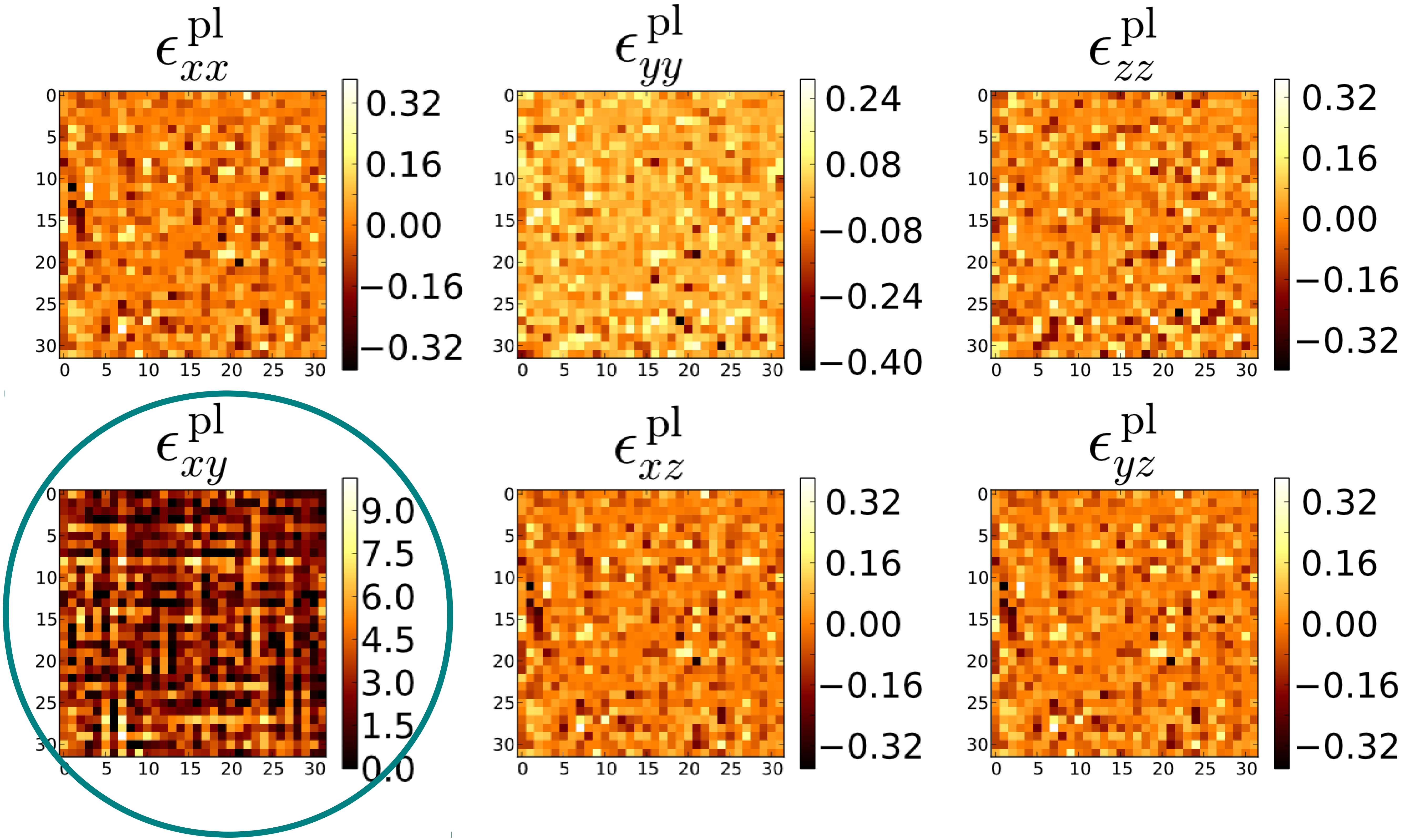
Yield: von Mises criterion, radial return
Simulate cube with L=32, periodic BCs
Same algorithm as 2d scalar model

Tensorial kernel

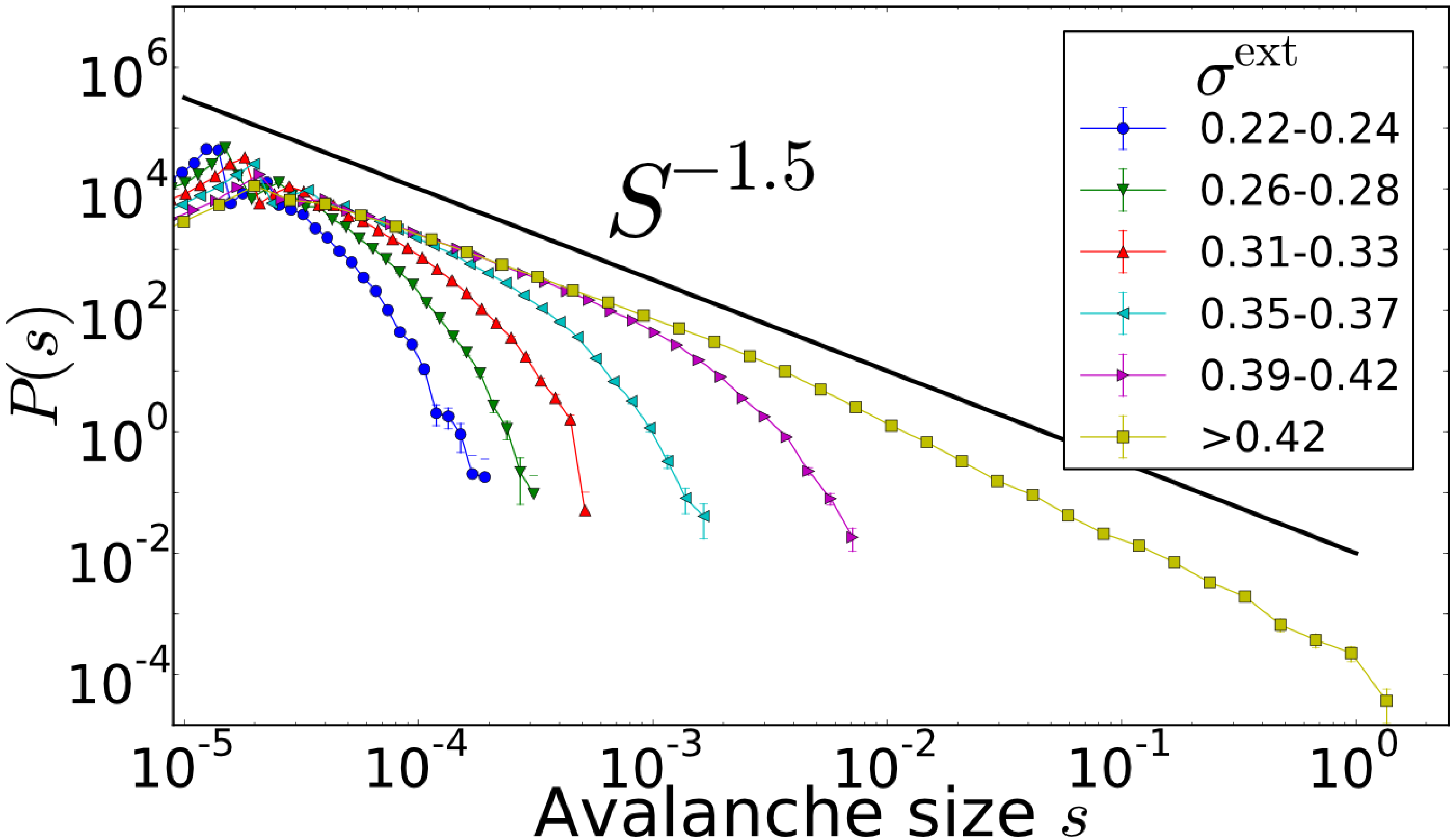
In 3d: $1/r^3$ decay, all components anisotropic

Courtesy S. Sandfeld

3d strain distributions (slice along z=0)



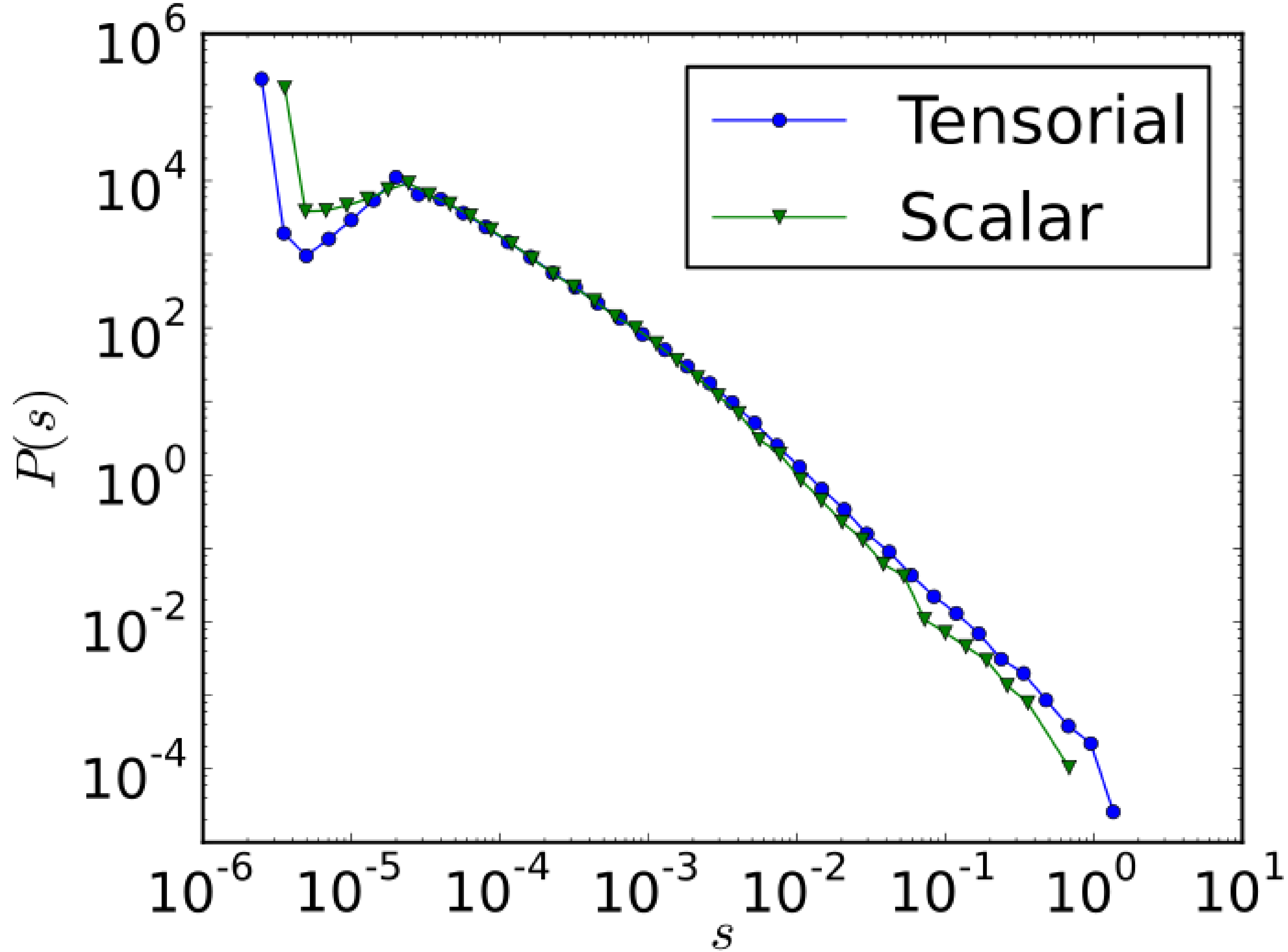
Avalanche size distributions – tensorial



Avalanche size=change in ε_{xy}

Exponent $\tau \lessapprox 1.5$

Avalanche sizes consistent with scalar model



Size distribution near depinning the same for tensorial and scalar models

c.f. Lin et al, arXiv:1403.6735

Conclusions

- Under shear loading, scalar model works well
- Depinning model for plasticity in amorphous materials gives power law distributed avalanches
- The universality class is not mean-field (a new universality class for depinning?)
- Mean-field behavior only occurs far from the transition
- Localization (which depends also on short-range interactions) induces crossover from mean-field
- Paper at: Phys. Rev. E 88, 062403 (2013)